The Theory of Temporary Equilibrium and the Keynesian Model

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1. Introduction

Starting with the analyses of Patinkin (1965, Ch. XIII) and, in particular, Clower (1965) and Barro/Grossman (1971), the idea of exchange under non-market clearing conditions — as opposed to the Walrasian assumption that exchange only takes place at market clearing prices — has attracted much attention in recent years[1]. Although the related literature deals to a large extent with Keynesian problems and also comes to Keynesian conclusions, the analytical framework usually employed is similar to that of ordinary general equilibrium models. Many crucial features of the Keynesian framework do not appear. Most[2] of the models do not employ the Keynesian consumption function and have a labour supply function which, unlike the Keynesian model, includes profit income as an argument. There is no model which really makes use of the Hicksian

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[1] For comprehensive discussions of the subject see the books by Barro/Grossman (1976), Malinvaud (1977) and Böhm (1980). A survey article on the earlier literature was written by Grandmont (1977).

[2] An exception is Korliras (1975 and 1978), who however, does not try to give a rationalization for his maintenance of the conventional behavioural equations.

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IS-LM curve analysis[3]; nowhere can you find the conventional aggregate demand curve relating commodity demand to the price level, and typically not even the transactions demand function for money has been retained[4].

Besides the difference in the formal structure between the extended Keynesian model and the new disequilibrium approaches, there is a difference in the emphasis placed on the microeconomic foundation of the behavioural equations. Whereas in Keynesian theory these equations tend to be rather ad hoc, in modern disequilibrium approaches they typically are derived from microeconomic optimization problems. So one might be tempted to believe that the optimization procedure is the source of the structural differences pointed out above. The present paper, however, casts considerable doubt on the validity of such a supposition.

On the following pages we shall derive a macroeconomic model of temporary equilibrium which has many features in common with the Keynesian-type macro model, but which is nevertheless based on micro-economic optimization behaviour. By studying its performance under different exogenous shocks we shall try to demonstrate that this model is a useful tool for explaining various types of temporary equilibria which might occur in the economy.

2. A Basic Outline of the Model

As suggested by Foley (1975) the model is specified in continuous time so that a clear distinction between stock and flow markets is possible. However, as explained in the next section, it also has essential features of a two-period model.

We depict an economy with three types of private agents, households, factories, and trading firms, where all agents within a group are identical. In addition there is also a rudimentary public sector.

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[3] A partial exception is the model of Barro and Grossman (1976, p. 149) which has an *IS* curve, though not the usual *LM* curve, for the special case of excess supply in both the labour and the commodity markets.

[4] Most of the models are of the one-asset variety where money is assumed to be a direct source of utility or the only store of value. Barro and Grossman (1976, pp. 109—113) do, however, present a two-asset version where money demand depends on private saving. This version coincides with the usual macroeconomic demand function if there is a unique relationship between private savings and the volume of production. A loanable funds version of the money demand function where the desired increase in money balances depends on the real wage rate acting "as a proxy for an income variable" can be found in Korliras (1975, p. 62).

Exchange takes place in four economic goods: an interest-bearing bond, labour, an all-purpose commodity, and money. With money as the medium of exchange there are accordingly three markets. The commodity market, though, may also be considered to consist of two submarkets, an intermediary market between factories and traders and a retail market. The behaviour of private agents is specified in such a way that the commodity and labour markets are flow markets and the bond market is a stock market. There exist no forward markets.

Households supply labour and own all bonds as well as both kinds of firms. In addition to their interest and wage income they receive all remaining profits. The income is used to buy consumption commodities from the retail market and to accumulate bonds. *Factories* produce the all-purpose commodity using capital and labour. The capital stock serves both as a direct factor of production in the usual sense, as well as an inventory. It is built up through commodity purchases in the retail market[5] and is financed by issuing bonds. Capital is a fixed factor at a given point in time but can, with the passage of time, gradually be changed through investments. *Traders* buy the output of factories and distribute it to final purchasers without maintaining commodity stores. Their rôle is to absorb the frictions in the economy's money circulation process. To this end they themselves need commodities from the retail market, i. e. from other traders, and hold a stock of money balances which they receive from households in exchange for bonds. The *government* buys commodities in the retail market in exchange for new money and distributes them as a public good to households. The public good enters into a separable household utility function and thus does not affect private behaviour. The government may also increase private money balances through stock transfers (helicopter money).

According to this specification of the model, three basic accounting identities in the economy are

$$Y \equiv C + I + H + G, \tag{1}$$

$$Y - H \equiv \Pi + wN, \tag{2}$$

$$\Pi \equiv \Pi^f + \Pi^t, \tag{3}$$

[5] This assumption is quite appropriate for investments in plants, but not so much for inventory investments. It would therefore be desirable to have a more extended model which explicitly allows for two kinds of capital goods, one bought at the market and the other self-produced.

where Y, C, I, H, and G denote national product (sales of the trading sector), consumption, investment, transaction cost and government expenditure; Π, w, and N denote aggregate profit income, wage rate, and employment; and Π^f and Π^t denote profit income from factories and trading firms. Profit income is defined to include interest income. All variables, except of course N, are measured in real terms, i. e., they are money values divided by the retail price. This implies that Y, C, I, H, and G also measure the numbers of the corresponding physical units produced by the factories.

The analysis is primarily restricted to the very short run. Strictly speaking, we examine the solution of the model for a given point in time. This solution is called temporary equilibrium. Section 5 adds a rudimentary analysis of the dynamic adjustment process towards Walrasian equilibrium.

The *temporary equilibrium* of the economy is defined such that each market is in one of two types of partial equilibria.

a) A fixed price equilibrium with quantity rationing. According to the principle of voluntary exchange the trading volume equals the lower of the quantity demanded and the quantity supplied.
b) An equilibrium in the usual sense where the market price has adjusted to equalize (effective) demand and supply.

We assume that the market rate of interest is able to change instantaneously, but that the nominal wage rate and the commodity price are sluggish; at most they can continuously adjust as time passes[6]. The trade margin is assumed to be fixed throughout. Accordingly, at each instant of time the bond market is in an equilibrium of type b), while the labour and commodity markets will generally be in an equilibrium of type a). This asymmetry is intended to capture the impression that the bond market is, in reality, by far the most perfect of the three markets, clearing on a daily basis[7].

Since 'disequilibrium' exchange is permitted in which either the demand or the supply side of the market is constrained, a rationing

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[6] A review of the literature which provides a microeconomic foundation of price rigidities is provided by Gordon (1976, pp. 207—211). We do not attempt to give an explanation for this empirically observable phenomenon in this paper.

[7] It is true that we can observe rationing in credit marktes. However, the reason seems to be the moral hazard problem involved with a particular kind of debtors rather than a temporary excess demand for credit due to the sluggishness of interest rates.

scheme must be imposed on the economy. We assume symmetry within a glass of agents, such that whenever a group of agents is constrained in a market all members of this group are equally constrained[8]. But we do not assume symmetry among different classes of agents. Instead, the rationing on the demand side of the commodity market is characterized by an *investment-last principle.* The idea behind this assumption is that firms use part of their capital as a buffer stock. In the case of excess demand in the commodity market, all kinds of non-investment demand are met first and firms invest less than they otherwise would have done, even if this means that investment becomes negative[9].

3. The Behaviour of Private Market Agents

The objective of a firm (factory or trading firm) is to maximize its market value, and of a household to maximize its utility. All private agents are price takers, but perceive their quantity constraints when trade is taking place. They plan for an infinite time horizon, yet divide time into two periods of different lengths, the present and the future. The 'present' is the period for which they believe current market signals will persist — say a year. The 'future' is the remainder, i. e., all future years. For simplicity, it is assumed that the planned levels of trade are invariant with respect to time both within the present and the future periods, although they may differ between these periods. All flow variables of the model are defined for a time interval which has the length of the current period.

Even though we assume that all market agents solve intertemporal optimization problems, we are not really interested in future planning variables. Instead we focus on the optimal behaviour for the current period.

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[8] This assumption is not very realistic for household employment. However, it seems to be relatively innocent. It can be shown that aggregate consumption demand is not affected by the way labour supply is rationed if the single houshold's utility function is Cobb-Douglas or a strictly increasing monotone transformation thereof (and if we maintain our assumption that all housholds are identical).

[9] Note that there is no limitation to the extent by which investments may become negative, provided, as we assume, that the stock of capital is strictly positive. Since investments are merely the time derivative of the capital stock, at a given point in time each finite level of disinvestments is possible without exhausting inventories.

Expectations about the future are held with certainty. Table 1 compares our assumptions concerning these expectations with those of Barro and Grossman (1976, Ch. 2).

Table 1. Expectations About the Future

	Constraints expected by households	Constraints expected by factories	Constraints expected by traders
This model	no	no	yes, exogenous constraints
Barro/ Grossman	no	no	—

	All agents' expected commodity price	All agents' expected real wage rate	All agents' expected interest rate	Real profit income expected by households
This model	equals current value	exogenous	exogenous	exogenous
Barro/ Grossman	equals current value	equals current value	equals current value	exogenous

The common idea behind our assumptions is that for the future period private agents expect the economy to be in a long-run equilibrium. In contrast with Barro and Grossman it is therefore assumed that not only real profit income but also the interest rate and the real wage rate are expected to obtain given 'natural' values. To assume in addition a natural commodity price level does not seem to be very realistic. Traders, who do not occur in the Barro-Grossman model, believe that they are always constrained since they are operating under increasing returns to scale, according to the Baumol-Tobin theory.

Our general procedure for determining a market agent's optimal choices will be governed by Clower's (1965) *dual decision hypothesis*, which later was generalized by Benassy (1975). Thus we assume that in choosing his optimal engagement in a particular market activity (i. e., in choosing his supply and demand in a particular market) an agent takes into account the constraints he faces in his other market activities, but not the constraint for the activity in question[10].

[10] This assumption implies that, in contrast to Walras' law, the excess demands which an agent shows in all markets do not necessarily add up to zero. Due to its specification in continuous time the model has two *separate* economy-wide budget constraints, one for flows and one for stocks. Since there are only two stocks that can be traded, namely money and bonds, the excess demands for these stocks add up to zero. (They

Since all agents within a sector have been assumed to be identical we economize space by adopting the familiar simplification that there is only one representative agent for each sector. Nevertheless we shall analyze his optimization problem as if he were an atomistic agent, too small to affect market parameters through his own decisions.

a) Households

We first study the behaviour of the representative household, using the following variables:

C, C^* ≡ real (physical) present and future consumption, $C, C^* \geq 0$;
B ≡ time budget per year, $B > 0$;
N, N^* ≡ present, future employment, $0 \leq N \leq B$, $0 \leq N^* \leq B$;
N_d ≡ labour demand of the factory sector, currently observed and expected for the present period, $N_d > 0$;
w ≡ real wage rate, currently observed and expected for the present period, $w > 0$;
w^* ≡ real wage rate expected for the future, $w^* > 0$;
r ≡ (real and nominal) momentary interest rate, currently observed and expected for the present period, $r > 0$;
r^* ≡ (real and nominal) momentary interest rate expected for the future, $r^* > 0$;
Π ≡ real profit income (including interest income), currently observed and expected to be earned during the present period, given the current stock of bonds[11], $\Pi > 0$;
Π^* ≡ real profit income (including interest income), expected to be earned in the future, given the current stock of bonds[11], $\Pi^* > 0$;
U ≡ utility function; $U : \mathbf{R}^4_+ \to \mathbf{R}$, $U_1, \ldots, U_4 > 0$, strictly quasi concave, twice continuously differentiable, positive income effects.

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would do this even if the rate of interest would not instantaneously drive each of them to zero.) However, apart from this, the flow excess demands for money, bonds, labour, and commodities generally do not balance. (Note, in addition, that for each single agent except traders the flow excess demand for money is zero since no stock of money is accumulated.)

[11] In planning its consumption path according to (4) the household implicitly plans to accumulate or decumulate its stock of bonds. Accordingly its expected profit income after the point in time where we observe the economy is a planning variable. Note, however, that *this* profit income is not measured by Π and Π^*. Instead, these variables give the profit income the household currently observes and *had* to expect if it neither saved nor dissaved.

Given the observed or expected market parameters r, r^*, w, w^*, Π, Π^* and N_d the representative household seeks to solve the following decision problem:

$$\max_{(C, C^*, N, N^*)} U(C, C^*, B-N, B-N^*) \tag{4}$$

s. t.[12]

$$\begin{aligned}(Bw+\Pi)\frac{1-e^{-r}}{r}+(Bw^*+\Pi^*)\frac{e^{-r}}{r^*}&\\ =[C+(B-N)w]\frac{1-e^{-r}}{r}+[C^*+(B-N^*)w^*]\frac{e^{-r}}{r^*},&\end{aligned} \tag{4a}$$

$$N \leq N_d, \tag{4b}$$

$$C, C^*, N, N^*, B-N, B-N^* \geq 0. \tag{4c}$$

In other words, the household tries to maximize its utility from consumption and leisure in the present and in the future under the three constraints; (a) that the discounted value of consumption and leisure [r. h. s. of (4a)] equals its wealth [l. h. s. of (4a)]; (b) that its employment cannot exceed the labour demanded; and (c) that consumption, leisure, and employment must not be negative. Note that, due to the investment-last principle, the household cannot in addition face a constraint at the commodity market[13].

Consider first the representative household's labour supply decision. Solving the optimization problem (4) disregarding constraint (4b) we find the optimal level of *labour supply*, N_s. In general it

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[12] With $a=\text{const.}$ and $r'(t)=\begin{cases} r^* & \text{for } t>1 \\ r & \text{for } t\leq 1\end{cases}$ it holds that

$$\int_0^1 a \exp\left(-\int_0^t r'(\tau)\,d\tau\right)dt = a\frac{1-e^{-r}}{r}$$

$$\text{and} \int_1^\infty a \exp\left(-\int_0^t r'(\tau)\,d\tau\right)dt = a\frac{1}{r^* e^r}.$$

[13] Thus we exclude Barro/Grossman's (1974; 1976, pp. 79—86) story of the frustrated consumer who reduces his labour supply because he faces a constraint on his consumption. Since it is difficult to find more than the occasional empirical example of waiting queues of frustrated consumers, this does not appear to be a deficiency of our approach. Barro/Grossman (1971, p. 91, fn. 18) refer to Russia, a good example of an economy which we do not want to model. Our assumption is in line with Howitt (1977, 1979) and Blinder (1978 a and b) who have criticized the Barro-Grossman effect with reference to inventory investments.

is given by a function of the type

$$N_s = N_s^0 (B, w, w^*, \Pi, \Pi^*, r, r^*), N_s^0 : \mathbf{R}^7_+ \to \mathbf{R}_+.$$

If the variables B, w^*, Π^*, and r^* are omitted, since they are exogenous not only to the household's decision problem, but also to the whole model, this function can be written as

$$N_s = N_s^1 (\underset{\binom{+}{0}}{w}, \underset{(?)}{r}, \underset{(-)}{\Pi}), N_s^1 : \mathbf{R}^3_+ \to \mathbf{R}_+, \quad (5)$$

where $N_s^1 \equiv N_s^0$ for all w, r, Π.

The signs given in parentheses beneath the argument of this function indicate the corresponding partial derivatives, where we assume that the nonnegativity constraints (4c) are not binding. Whereas the influence of Π on labour supply is due to a clear-cut income effect, the rôles played by the wage rate and the interest rate are theoretically ambiguous because of the well-known counteraction of income and substitution effects. Empirically, however, there is some evidence that for the wage rate the substitution effect dominates[14]. Thus we assume $\partial N_s^1/\partial w \geq 0$.

In addition to the labour supply it will be useful to consider the household's *optimal feasible employment*, $\hat{N}$, which is the solution of (4) for N, where constraint (4b) is *not* omitted. Because of the strict quasi concavity of U this variable is given by[15]

$$\hat{N} = \min (N_s, N_d). \quad (6)$$

Next we analyse the representative household's *consumption* demand, C_d, which is the solution of (4) for C. If we want to proceed along usual lines we may write consumption demand as a function of the type

$$C_d = C_d' (B, w, w^*, \Pi, \Pi^*, r, r^*, N_d), C_d' : \mathbf{R}^8_+ \to \mathbf{R}_+,$$

or, suppressing the same variables as before,

$$C_d = C_d'' (w, \Pi, r, N_d), C_d'' : \mathbf{R}^4_+ \to \mathbf{R}_+,$$

where $\partial C_d/\partial N_d = 0$ for $N_d > N_s$ and $C_d'' \equiv C_d'$ for all w, Π, r, N_d.

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14 Cf., e. g., Ashenfelter and Heckman (1974).

15 Optimal feasible employment is a pure planning variable. We shall later see that in a temporary equilibrium it equals effective employment. Without the strict quasi concavity of U it would be possible that $\hat{N} < N_d$ if $N_s > N_d$.

However, it will turn out to be helpful to derive optimal consumption demand in a somewhat different way.

Consider the optimization problem

$$\max_{(C,\, C^*,\, N^*)} U|_{N=x} \quad \text{s. t. (4 a) and (4 c),} \tag{7}$$

where x, $0 \leq x \leq B$, is an arbitrarily given employment level. Let $\hat{C}_d$ be the solution of this problem for C. $\hat{C}_d$ is *optimal conditional consumption*, conditional since it indicates the household's best choice if employment is predetermined at x. In general $\hat{C}_d$ is given by a function of the type

$$\hat{C}_d = C_d^0 \,(\Pi + wx,\, B,\, w^*, \Pi^*,\, r,\, r^*,\, x),\ \ C_d^0 : \mathbf{R}^7_+ \to \mathbf{R}_+,$$

or, suppressing constants, by

$$\hat{C}_d = C_d^1 \,(\underset{(+)}{\Pi + wx},\, \underset{(?)}{r},\, \underset{(?)}{x}),\ \ C_d^1 : \mathbf{R}^3_+ \to \mathbf{R}_+ \tag{8}$$

where $C_d^1 \equiv C_d^0$ for all Π, w, x, and r. Note that Π and w do not have to appear separately in this function. Inspection of the budget constraint (4a) shows that nothing in the decision problem alters if, given x, Π and w change adversely without affecting the sum $\Pi + wx$. The parentheses beneath the arguments of the function indicate what the preceding assumptions imply for the corresponding partial derivatives if, as we assume, the non-negativity constraints (4c) are not binding.

By construction the conditional consumption function has the property[16]

$$C_d^1 \,(\Pi + w\hat{N},\, r,\, \hat{N}) = C_d'' \,(w, \Pi, r, N_d) \quad \text{for all } \ \Pi, w, r, N_d.$$

Thus, to determine the household's optimal consumption, we can

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[16] Since it might not be obvious to the reader that this equality holds if $\hat{N} = N_s < N_d$, i. e., if the household is not effectively constrained, this footnote gives a formal proof. Let C_d, C_d^*, N_s, N_s^* be the solutions of (4) for C, C^*, N, N^*. Furthermore, let $\hat{C}_d$, $\hat{C}_d^*$, $\hat{N}_s^*$ be the solutions of (7) for C, C^*, N^* if $x = N_s$. Suppose $C_d \neq \hat{C}_d$. Then, by definition of C_d, C_d^*, N_s, N_s^* and the strict quasi concavity of U we have $U(C_d, C_d^*, N_s, N_s^*) > U(\hat{C}_d, \hat{C}_d^*, N_s, \hat{N}_s^*)$. On the other hand, by definition of $\hat{C}_d$, $\hat{C}_d^*$, $\hat{N}_s^*$ and the strict quasi concavity of U we have $U(\hat{C}_d, \hat{C}_d^*, N_s, \hat{N}_s^*) > U(C_d, C_d^*, N_s, N_s^*)$, a contradiction. Thus, $C_d = \hat{C}_d$, Q. E. D.

proceed in two steps. First we calculate its optimal labour supply N_s as given by (5). Then we compare this value with the labour demand N_d facing the representative household. The lower of these two values is the household's optimal feasible employment. Inserting the optimal feasible level of employment into the conditional consumption function (8) for x we achieve the solution of problem (4) for C, i. e.,

$$C_d = C_d^1 (\underset{(+)}{\Pi + w\hat{N}}, \underset{(?)}{r}, \underset{(?)}{\hat{N}}). \tag{9}$$

b) Factories

Let us now consider the behaviour of the representative factory. In addition to the information given in the previous section we use the following definitions:

V^f ≡ real market value of the representative factory at the beginning of the first period;

Π^f, Π^{f*} ≡ real present, future profits of the representative factory (including interest payments to households);

N_s ≡ labour supply of the household sector, currently observed and expected for the first period, $N_s > 0$;

I ≡ present real (physical) investment in plant and inventory;

I_s ≡ real (physical) commodity supply in the retail market, presently available for investment and expected to be available during the first period (after serving all other kinds of demand);

K, K^* ≡ real (physical) present, future capital stock, $K > 0$, $K^* \geq 0$;

Ω ≡ production function; $\Omega : \mathrm{R}^2_+ \to \mathrm{R}_+$; strictly concave; twice continuously differentiable; $\Omega_1, \Omega_2 > 0$; $\Omega_{12} = \Omega_{21} > 0$; $\lambda^\beta \Omega(x, y) = \Omega(\lambda x, \lambda y)$, $x, y > 0$, $0 < \beta < 1$;

α ≡ constant trade margin as a share of the retail price, $0 < \alpha < 1$;

Y, Y^* ≡ present, future physical output of the representative factory ≡ present, future real and physical output of the trading sector ≡ present, future real and physical national product[17], $Y, Y^* \geq 0$;

[17] Since we define a real variable to be its nominal counterpart divided by the retail price, the real output of the representative factory is its nominal sales divided by the retail price, i. e., $(1-\alpha)\, Y$.

Y_d^t $\equiv$ physical commodity demand of the trading sector, currently observed and expected for the present period, $Y_d^t > 0$.

Given the observed or expected market parameters[18] r, r^*, w, w^*, N_s, I_s, Y_d^t and given the initial stock of capital, K, the representative factory faces the following problem:

$$\max_{(N, N^*, I)} V^f = \Pi^f \frac{1-e^{-r}}{r} + \Pi^{f*} \frac{e^{-r}}{r^*} - K - I \frac{1-e^{-r}}{r} \tag{10}$$

s. t.

$$\Pi^f = (1-\alpha)\, Y - wN, \; \Pi^{f*} = (1-\alpha)\, Y^* - w^* N^*, \tag{10a}$$

$$Y = \Omega\,(K, N), \;\; Y^* = \Omega\,(K^*, N^*), \tag{10b}$$

$$K^* = K + I, \tag{10c}$$

$$N, N^*, Y, Y^*, K^* \geq 0, \tag{10d}$$

$$N \leq N_s, \;\; Y \leq Y_d^t, \; I \leq I_s. \tag{10e}$$

Eqs. (10) and (10a—d) describe the decision problem of a factory in Walrasian equilibrium: It must choose the present and the future employment levels, N and N^*, as well as the change in the capital stock, I, which maximize its market value. Condition (10e) characterizes the new element entering the decision problem when we allow for rationing, where labour supply, output demand, and investment supply may be lower than the factory's demand for labour, supply of output, or demand for investment, respectively. We assume that the firm's optimum is characterized by $\Pi^f > 0$.

Solving (10) for I, disregarding the constraint $I \leq I_s$, we get the optimal level of *investment demand*, I_d. In general this is a function of the form

$$I_d = I_d^0\,(r, r^*, w^*, K, \alpha), \; I_d^0 : \mathbf{R}_+^5 \to \mathbf{R}_+,$$

but, if we suppress constants, we have

$$I_d = I_d^1\,\underset{(-)}{(r)}, \; I_d^1 : \mathbf{R}_+ \to \mathbf{R}, \tag{11}$$

with $I_d^1 \equiv I_d^0$ for all r. A footnote details the derivation of this

[18] Recall that the representative firm is assumed to behave as if it could not affect market parameters. Although a single large firm would definitely be able, e. g., to alter I_s through its own commodity supply to the trading sector and Y_d^t through its own investment demand in the retail market, an atomistic firm could not affect these parameters.

function[19]. It is assumed that there exists $x > 0$ such that $I_d > 0$ for $r < x$.

Next we consider the factory's labour demand and commodity supply. Solving (10) for N and Y, disregarding the constraints $N \leq N_s$ and $Y \leq Y_d^t$, we get what Clower has called *notional labour demand*, $\tilde{N}_d$, and *notional commodity supply*, $\tilde{Y}_s$. Define the partial production function Φ: $\mathbf{R}_+ \rightarrow \mathbf{R}_+$ with

$$\Phi(x) \equiv \Omega(x, K),\ \Phi' = \Omega_1 > 0,\ \Phi'' = \Omega_{11} < 0, \quad \text{for all} \quad x \geq 0, \qquad (12)$$

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[19] From the marginal conditions $(1-\alpha)\,\Omega_2(K^*, N^*) - w^* = 0$ and $-(1-e^{-r})/r + (1-\alpha)\,\Omega_1(K^*, N^*)/(r^* e^r) = 0$ we calculate

(I) $dI/dr = -n/d$, where $n \equiv [e^r(1-r) - 1]/r^2$ and

$d \equiv [(1-\alpha)/r^*]\,(\Omega_{11} + b\Omega_{12}N^*/K^*),\ b \equiv -K^*\Omega_{21}/(N^*\Omega_{22}) > 0.$

Since $e^r > 0$, we have

$$\operatorname{sgn} n = \operatorname{sgn}[(1-r) - e^{-r}].$$

Note that

(II) $(1-r) = e^{-r} = 1$, if $r = 0$,

and

(III) $\dfrac{\partial(1-r)}{\partial r} = \dfrac{\partial e^{-r}}{\partial r} = -1$, if $r = 0$.

Since $(1-r)$ is a linear function of r and e^{-r} is strictly convex, (II) and (III) imply $e^{-r} > 1 - r$ and consequently

(IV) $n < 0$, if $r \neq 0$.

The sign of the denominator (d) can be established as follows: For a production function homogeneous of degree β we have $\lambda^{\beta-1}\,\Omega_1(K^*, N^*) = \Omega_1(\lambda K^*, \lambda N^*)$. Since $0 < \beta < 1$, differentiation with respect to λ at $\lambda = 1$ gives $(\beta - 1)\,\Omega_1 = \Omega_{11}K^* + \Omega_{12}N^* < 0$ or

(V) $\Omega_{11} + \Omega_{12}N^*/K^* < 0$.

Analogously, $(\beta-1)\,\Omega_2 = \Omega_{21}K^* + \Omega_{22}N^* < 0$. This implies $\Omega_{21}K^*/(\Omega_{22}N^*) + 1 > 0$, or

(VI) $b < 1$.

Obviously (V) and (VI) ensure $d < 0$. So, with respect to (IV) and (I), we find

$$\frac{dI}{dr} < 0, \text{ if } r \neq 0.$$

where K is a constant. Then, obviously, notional labour demand is determined by the marginal condition

$$\Phi' (\tilde{N}_d) = \frac{w}{1-\alpha},$$

if, as we assume, the constraint $N \geq 0$ is not binding. Suppressing the constant α we can also write

$$\tilde{N}_d = \tilde{N}_d^0 \underset{(-)}{(w)}, \ \tilde{N}_d^0 : \mathbf{R}_+ \to \mathbf{R}_+. \tag{13}$$

Notional commodity supply is accordingly

$$\tilde{Y}_s = \underset{(+)}{\Phi} (\tilde{N}_d). \tag{14}$$

Note that, while the labour demand and commodity supply decisions are intimately linked to each other, both together are obviously separable from the investment decision.

The effective counterparts of the notional variables given in (13) and (14) are calculated by referring to the dual decision hypothesis. *Effective labour demand*, N_d, is the solution of (10) for N, disregarding the constraint $N \leq N_s$. Since the firm knows that the level of commodity demand is given by Y_d^t, and because of the concavity of Φ[20], its effective labour demand is

$$N_d = \min [\tilde{N}_d, \underset{(+)}{\Phi^{-1}} (Y_d^t)]. \tag{15}$$

Effective commodity supply, Y_s, is the solution of (10) for Y, disregarding the constraint $Y \leq Y_d^t$. Since the firm knows that the level of labour supply is N_s, it is unable to supply more output than it can produce with this labour supply. Thus, with respect to $\Omega'' < 0$, we find

$$Y_s = \min [\tilde{Y}_s, \underset{(+)}{\Phi} (N_s)]. \tag{16}$$

c) Traders

To analyse the problem of the representative trading firm we add the following definitions to those provided in the previous two sections:

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[20] Without concavity of Φ there could be multiple profit maxima such that $N_d < \Omega^{-1} (Y_d^t) < \tilde{N}_d$ would be possible. A similar remark applies to (16).

V^t $\equiv$ real market value of the representative trading firm at the beginning of the first period;

Y_s $\equiv$ physical commodity supply of the factory sector, currently observed and expected for the present period, $Y_s > 0$;

Y_s^* $\equiv$ expected future physical commodity supply of the factory sector, $Y_s^* > 0$;

Y_d $\equiv$ real (physical) commodity demand in the retail market, currently observed and expected for the present period, $Y_d > 0$;

Y_d^* $\equiv$ expected future real (physical) commodity demand in the retail market, $Y_d^* > 0$;

Π^t, Π^{t*} $\equiv$ real present, future profits (including interest payments to households) of the representative trading firm;

M, M^* $\equiv$ real present, future money balances, $M, M^* \geq 0$;

a $\equiv$ transaction-cost parameter, $a > 0$;

H, H^* $\equiv$ real (physical) present, future transaction costs, $H, H^* \geq 0$.

Given the observed or expected market parameters[21] r, r^*, Y_s, Y_s^*, Y_d, and Y_d^*, the representative trading firm has to solve the following problem:

$$\max_{(M, M^*, Y, Y^*)} \quad V^t = \Pi^t \frac{1-e^{-r}}{r} + \Pi^{t*} \frac{e^{-r}}{r^*} - M - (M^* - M)\, e^{-r} \tag{17}$$

s. t.

$$\Pi^t = \alpha Y - H, \; \Pi^{t*} = \alpha Y^* - H^*, \tag{17a}$$

$$H = aY/M, \; H^* = aY^*/M^*, \tag{17b}$$

$$M, M^*, Y, Y^* \geq 0, \tag{17c}$$

$$Y \leq Y_d, \; Y^* \leq Y_d^*, \; Y \leq Y_s, \; Y^* \leq Y_s^*. \tag{17d}$$

Accordingly it tries to choose the stocks of real money balances for the present and the future as well as the present and future

[21] By manipulation of its own transaction costs a single trading firm would of course be able to change Y_d and Y_d^*, i. e., the demand it faces in the retail market. But again we have to recall the definition of the representative agent. An atomistic trading firm which, as we assumed in section 2, is forced to buy the commodities it needs for its transactions from other traders could not possibly alter the demand for its own output.

transactions volumes as measured by Y and Y^* in such a way that its market value is maximized. We assume that in the optimum

$$\Pi^t \frac{1-e^{-r}}{r} - M\,(1-e^{-r}) = (\Pi^t - rM)\frac{1-e^{-r}}{r} > 0.$$

The transaction-cost formula given in (17b) may be interpreted as the Baumol-Tobin function where $2a$ measures a fixed cost per transaction[22]. Define a function $\hat{V}^t : \mathbf{R}_+ \to \mathbf{R}$ such that $\hat{V}^t(Y) = \max_{(M, M^*, Y^*)} V^t$ s. t. (17 a—c) and $Y^* \le Y_d^*$, $Y^* \le Y_s^*$. Since the transaction-cost function ensures that traders enjoy increasing returns to scale and since in the optimum of the whole problem (17) we have $(\Pi^t - rM)\frac{1-e^{-r}}{r} > 0$, the function $\hat{V}^t$ has the property

$$\frac{d\hat{V}^t}{dY} > 0 \quad \text{for all} \quad Y > 0. \tag{18}$$

Because of the dual decision hypothesis, the trading firm's optimal *commodity supply*, Y_s^t, in the current period is the solution of (17) for Y, disregarding the constraint $Y \le Y_d$, and its optimal *commodity demand*, Y_d^t, is the solution of (17) for Y, disregarding the constraint $Y \le Y_s$. Because of (18) we find

$$Y_s^t = Y_s \tag{19}$$

and

$$Y_d^t = Y_d. \tag{20}$$

Thus, the trader simply transmits the commodity supply by factories, Y_s, to the final market and the commodity demand from the final market, Y_d, to factories, without himself trying to manipulate the market signals. For this reason we shall henceforth no longer distinguish between Y_s^t and Y_s on the one hand and Y_d^t and Y_d on the other. Instead, we shall use Y_d for (real and physical) commodity demand and Y_s for (real and physical) commodity supply in general.

Let us now define the representative trader's *optimal feasible trading volume*, $\hat{Y}$, as the solution of (17) for Y. Because of (18)—(20) we obviously have

$$\hat{Y} = \min(Y_d, Y_s). \tag{21}$$

[22] The number of transactions per period is $Y/(2M)$ where M denotes the average and $2M$ the initial stock of real money balances.

Provided with this variable, we can easily calculate the representative trading firm's current *demand for real money balances*, M_d, which is the solution of (17) for M, and its *planned real transaction costs*, H_d, which is the solution of (17) for H. We find the familiar square root formulas:

$$M_d = \sqrt{a\hat{Y}/r} \equiv M_d^1 \underset{(+)\,(-)}{(\hat{Y}, r)}, \quad M_d^1 : \mathbf{R}_+^2 \to \mathbf{R}_+, \tag{22}$$

for all $\hat{Y}$ and r, and a = const., and

$$H_d = \sqrt{a\hat{Y}r} \equiv H_d^1 \underset{(+)\,(+)}{(\hat{Y}, r)}, \quad H_d^1 : \mathbf{R}_+^2 \to \mathbf{R}_+, \tag{23}$$

for all $\hat{Y}$ and r, and a = const.

4. The Model Structure in Temporary Equilibrium

This section develops a macroeconomic model of temporary equilibrium. In addition to the behavioural and definitional equations established so far, we first list some further equations of the model. Then we try to combine the various pieces of information in such a way that we obtain a model structure as similar as possible to that of the (neo) Keynesian model.

According to the basic accounting identity (1) total commodity demand is

$$Y_d \equiv C_d + I_d + H_d + G_d, \tag{24}$$

where G_d denotes the exogenous level of real (physical) government demand, $G_d \geq 0$.

Among the conditions of a temporary equilibrium in the economy we have

$$\frac{M^n}{p} = M_d, \tag{25}$$

$$N = \min\,(N_d, N_s), \tag{26}$$

$$Y = \min\,(Y_d, Y_s), \tag{27}$$

$$H_d = H, \quad C_d = C, \quad G_d = G, \tag{28}$$

where $M^n \equiv pM > 0$ is the nominal stock of money balances owned by the private sector and $p > 0$ the commodity price in terms of money (retail price). (25) is the usual condition for a stock equilibrium in the bond(-money) market if the market rate of interest is fully flexible as we assume. (26) and (27) characterize equilibria with quantity rationing for the labour and commodity markets. (28) is an implication of the investment-last principle.

Finally the production technology implies a one-to-one relationship between national product and employment which must hold under all circumstances:

$$Y = \Phi(N). \tag{29}$$

a) The *LM* Curve

A comparison between (21) and (27) shows that in a temporary equilibrium the optimal feasible trading volume equals national product:

$$\hat{Y} = Y. \tag{30}$$

Substituting Y for $\hat{Y}$ in (22) and using (25) we thus obtain

$$\frac{M^n}{p} = M_d^1 \underset{(+)\,(-)}{(Y, r)}, \tag{31}$$

or solving for r,

$$r = r^0 \Big(\underset{(-)}{\frac{M^n}{p}}, \underset{(+)}{Y}\Big), \; r^0 : \mathbf{R}_+^2 \to \mathbf{R}_+. \tag{32}$$

This equation describes the usual upward sloping *LM* curve as illustrated in Fig. 1α. Any kind of temporary equilibrium is necessarily characterized by a point on this curve[23].

b) The Consumption Function

Using the basic accounting identity (2) one can write the consumption function (9) as

$$C_d = C_d^1 \,[\underset{(+)}{Y - H + w(\hat{N} - N)}, \underset{(?)}{r}, \underset{(?)}{\hat{N}}].$$

A comparison between (6) and (26) shows that in a temporary equilibrium optimal feasible employment equals actual employment:

$$\hat{N} = N. \tag{33}$$

Thus, in such a situation, the consumption function reduces to

$$C_d = C_d^1 \,(Y - H, r, N).$$

---

[23] Using (22) we find the explicit version $r^0 (M^n/p, Y) = aY/(M^n/p)^2$ indicating that the *LM* curve is a straight line through the origin.

Now, (29) implies that N is strictly linked to Y; furthermore (23), (28), and (30) imply that in a temporary equilibrium H depends on Y and r. Accordingly we have

$$C_d = C_d^1 \left[Y - H_d^1 (Y, r), r, \Phi (Y)\right],$$

or more concisely,

$$C_d = \underset{(+)\;(?)}{C_d^2 (Y, r)}, \quad C_d^2 : \mathbf{R}_+^2 \rightarrow \mathbf{R}_+, \tag{34}$$

where $C_d^2 \equiv C_d^1$ for all Y and r. With the indicated sign for the marginal rate of consumption we assume that the overall impact of income on consumption is positive (cf. fn. 24).

The consumption function C_d^2 is not a behavioural function in the usual sense. Instead it gives a relationship between consumption demand, national product, and the rate of interest that generally only holds in a temporary equilibrium. Nevertheless C_d^2 contains the important information that the reason why Y and r have particular values in the temporary equilibrium does not by itself influence consumption demand. A priori, one might, e. g., suppose that it makes a difference whether a certain level of national product is achieved while households are constrained in the labour market or whether it is achieved while they are unconstrained. But given the representative household's utility function $U(C, C^*, B-N, B-N^*)$ this supposition is wrong. If we wanted to allow household behaviour to be affected by the frustration about the bare fact of being constrained we would have to add further arguments to the utility function.

c) The *IS* Curve

Inserting (11), (34), and (23) into (24), where, with regard to (23), we once again utilize (30), we find that in any kind of temporary equilibrium commodity demand is given by

$$Y_d = \underset{(-)}{I_d^1 (r)} + \underset{(+)\;(?)}{C_d^2 (Y, r)} + \underset{(+)\;(+)}{H_d^1 (Y, r)} + G_d. \tag{35}$$

We make use of this expression analogously to the Keynesian models. Consider the *unconstrained level of commodity demand*, $\overline{Y}_d$, which is defined as the solution of (35) for Y_d under the condition $Y_d = Y$. This variable is given by a function of the type

$$\overline{Y}_d = \underset{(-)\;(+)}{\overline{Y}_d^0 (r, G_d)}, \quad \overline{Y}_d^0 : \mathbf{R}_+^2 \rightarrow \mathbf{R}_+, \tag{36}$$

which describes the *IS* curve as shown in Fig. 1α. To establish the signs indicated beneath the arguments of this function we assume that[24]

$$0 < \partial C_d^2/\partial Y + \partial H_d^1/\partial Y < 1 \quad \text{and} \quad \partial I_d^1/\partial r + \partial C_d^2/\partial r + \partial H_d^1/\partial r < 0.$$

The *IS* curve we derived is similar, but not identical to the usual *IS* curve of the Keynesian model. The *IS* curve normally represents a necessary and sufficient condition for a partial equilibrium in the commodity market with unconstrained demand independent of other markets. The curve described by (36) does not have this restricted meaning. Instead, it represents a set of necessary conditions for a temporary equilibrium in the whole economy for the case where commodity demand is not effectively constrained. (Thus the term "*IS* curve" understates the number of equilibrium conditions required).

d) The Commodity Demand Curve

As usual in neo-Keynesian economics we obtain the aggregate commodity demand curve, figuratively speaking, by finding how the intersection point of the *LM* and *IS* curves shifts under a change in the commodity price level. Inserting (32) into (36) and solving for $Y = \overline{Y}_d$ one obtains the corresponding formal expression

$$\overline{Y}_d = \overline{Y}_d^1 \Big(\underset{(+)}{\frac{M^n}{p}}, \underset{(+)}{G_d} \Big), \quad \overline{Y}_d^1 : \mathbf{R}_+^2 \to \mathbf{R}_+, \tag{37}$$

where the indicated signs are obvious. Given the government's control variables M^n and G_d, (36) describes the familiar downward sloping curve in a (p, Y) diagram as illustrated in Fig. 1[25].

[24] To ensure that these plausible assumptions are compatible with previous assumptions of the model, a special example with $U(C, C^*, B-N, B-N^*) = C^\alpha C^{*\beta} (B-N)^\gamma (B-N^*)^\delta$, $\alpha = \beta = \gamma = \delta > 0$, was calculated. The example implies that by choosing the transaction-cost parameter a sufficiently small $\partial C_d^2/\partial Y + \partial H_d^1/\partial Y$ approaches as closely as we wish the value of 1/3 and $\partial C_d^2/\partial r + \partial H_d^1/\partial r$ approaches as closely as we wish to $[e^r(1-r)-1]/r^2$, an expression which in fn. 19 was shown to be negative for $r \neq 0$. Since the utility function of the example is separable these results are independent of the shape of the production function.

[25] From fn. 23 we know that the *LM* curve is described by $r = aY/(M^n/p)^2$. By assumption we have $M^n, p, a, C_d > 0$, $G_d \geq 0$. Also by assumption there exists $x > 0$ such that $I_d > 0$ for $r < x$. In connection with $H_d = \sqrt{a\hat{Y}r}$ from (23) this implies that there exists an intersection point of the *IS* and *LM* curves for strictly positive r and Y. Thus $\overline{Y}_d > 0$.

By construction the function $\bar{Y}_d{}^1$ only gives the unconstrained level of commodity demand, $\bar{Y}_d$, and not consumption demand in general, Y_d, the variable we are primarily interested in. Fortunately, however, $\bar{Y}_d$ has quite appealing properties in a situation of temporary equilibrium:

$$\bar{Y}_d \leq Y_s \Rightarrow Y_d = \bar{Y}_d = Y, \tag{38}$$

$$\bar{Y}_d > Y_s \Rightarrow Y_d > Y_s = Y. \tag{39}$$

These properties ensure that $Y = \min(Y_s, \bar{Y}_d)$. Thus, if we want to determine national product, $\bar{Y}_d$ represents Y_d perfectly. The only deficiency of $\bar{Y}_d$ is that in the case $\bar{Y}_d > Y_s$ we may have $\bar{Y}_d \neq Y_d$. Yet, according to (39) in this case $\bar{Y}_d$ is still a correct indicator of the prevalence of excess demand as such. The proofs for (38) and (39) are given in the appendix.

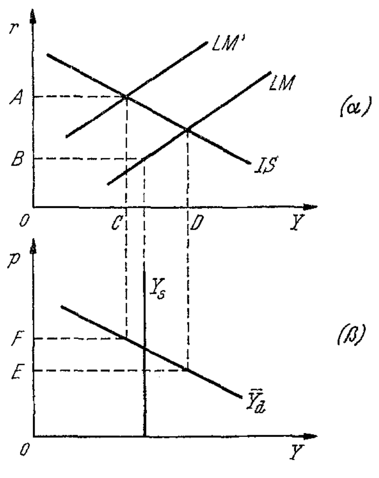

Fig. 1. The commodity demand curve

The discussion up to this point is summarized in Fig. 1. This figure shows the aggregate demand curve (37) (diagram β) together with the corresponding *IS* and *LM* curves (diagram α) and illustrates how an increase in the price level by *EF* shifts the *LM* curve from *LM* to *LM'*, thereby inducing a reduction of $\bar{Y}_d$ by *CD*. Fig. 1 also contains an aggregate supply curve, Y_s, so that the corresponding changes in national product and the rate of interest can be demonstrated. If the price level is OE in diagram β, we have $\overline{Y}_d > Y_s$ and thus $Y_d > Y_s = Y$ and, according to (32), the rate of interest is OB. If the price level rises of OF, national product decreases to OC and the rate of interest rises to OA. Because $\overline{Y}_d = Y_d = Y < Y_s$ both variables are now determined in the usual way by the intersection of the IS and LM curves.

e) The Labour Supply Curve

Using the basic accounting identity (2) one can write the labour supply function (5) as

$$N_s = N_s^1 \underset{\binom{0}{+}}{(w}, \underset{(?)}{r}, \underset{(-)}{Y - wN - H}).$$

For the case of a temporary equilibrium this expression becomes

$$N_s = N_s^1 \left\{ w, r^0 \left[\frac{M^n}{p}, \Phi(N) \right], \right. \tag{40}$$
$$\left. \Phi(N) - wN - H_d^1 \left[\Phi(N), r^0 \left(\frac{M^n}{p}, \Phi(N) \right) \right] \right\}$$

where (32), (29), (23), (28), and (30) have been used. Thus, labour supply eventually depends on the real wage rate, w, the real stock of money balances, $\frac{M^n}{p}$, and employment, N.

A comparison between (40) and (35) indicates that the rôle of employment in determining labour supply is formally similar to that of national product in determining commodity demand. For this reason we may proceed analogously to above and define a variable, called the *unconstrained level of labour supply*, $\overline{\overline{N}}_s$. $\overline{\overline{N}}_s$ is the solution of (40) for N_s under the condition $N_s = N$. It is given by a function $\overline{\overline{N}}_s^0 : \mathrm{R}_+^2 \to \mathrm{R}_+$ with

$$\overline{\overline{N}}_s = \overline{\overline{N}}_s^0 \left(\underset{(+)}{w}, \underset{(?)}{\frac{M^n}{p}} \right) \text{ for } \overline{\overline{N}}_s \leq \tilde{N}_d. \tag{41}$$

The sign for the influence of the real wage rate is obtained from

$$\frac{\partial \overline{\overline{N}}_s^0}{\partial w} = - \frac{\left. \frac{d(N - N_s)}{dw} \right|_N}{\left. \frac{d(N - N_s)}{dN} \right|_w} > 0 \text{ at } N = N_s = \overline{\overline{N}}_s \leq \tilde{N}_d, \tag{42}$$

where

$$\left.\frac{d\,(N-N_s)}{dw}\right|_N = -\frac{\partial N_s^1}{\partial w} + \frac{\partial N_s^1}{\partial \Pi}\, N < 0 \tag{42a}$$

and

$$\left.\frac{d\,(N-N_s)}{dN}\right|_w = 1 - \frac{\partial N_s^1}{\partial r}\frac{\partial r^0}{\partial Y}\,\Phi' - \frac{\partial N_s^1}{\partial \Pi}\,[\Phi'\,(1-\alpha) - w + A] > 0 \tag{42b}$$

for $N \leq \tilde{N}_d$, $A \equiv \Phi' \left(\alpha - \frac{\partial H_d^1}{\partial Y} - \frac{\partial r^0}{\partial Y}\frac{\partial H_d^1}{\partial r}\right)$.

The trade margin α, $0<\alpha<1$, has been employed in these formulations for reasons that will become clear below. Because of the signs of the partial derivatives with respect to w and Π as given in (4) we obviously have $\partial \bar{\bar{N}}_s^0/\partial w > 0$ if (a) $(\partial N_s^1/\partial r)(\partial r^0/\partial Y)\,\Phi' < 1$, (b) $\Phi'\,(N)\,(1-\alpha) \geq w$, and (c) $A>0$. Condition (a) is assumed to hold. Suppose employment increases by a unit. Then output will increase, raising the rate of interest via an increase in the transactions demand for money. The increase in the rate of interest might possibly increase labour supply. That this induced increase in labour supply is less than a unit is the content of the assumption[26]. Condition (b) requires that the marginal revenue product of labour does not fall short of its price. By the strict concavity of the production function and by the definition of notional labour demand, $\tilde{N}_d$, the latter is the case if $N \leq \tilde{N}_d$. (Because of $N_d \leq \tilde{N}_d$ from (15) and $N = \min\,(N_s, N_d)$, the condition $N \leq \tilde{N}_d$ in turn allows for all employment levels that may occur in a temporary equilibrium.) Concerning condition (c) it is shown in a footnote that in a temporary equilibrium we have $A = \frac{\Phi'}{Y}\left[\alpha Y - \frac{1}{2}\,(H_d - rM_d)\right]$. The expression is strictly positive, since by a previous assumption the trading sector enjoys profits strictly greater than the inputed interest costs for the stock of money employed, $\alpha Y - H_d > rM_d$[27].

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26 It is easy to verify by means of a special example that condition (a) is compatible with our assumptions about the household utility function, the transaction-cost function, and the production function.

27 Given that $\partial r^0/\partial Y = -\frac{\partial M_d^1/\partial Y}{M_d^1/\partial r}$, using the explicit functions for M_d^1 and H_d^1 as given in (22) and (23) and substituting $\hat{Y} = Y$ according to (30) we find

$$A = \Omega' \left[\alpha - \frac{1}{2}\frac{1}{Y}\sqrt{arY} - \frac{r}{Y}\frac{1}{2}\sqrt{aY/r}\right]$$

$$= \frac{\Omega'}{Y}\left[\alpha Y - \frac{1}{2}\,(H_d - rM_d)\right].$$

We have been able to establish the properties of $\bar{\bar{N}}_s{}^0$ only for $\bar{\bar{N}}_s \leq \tilde{N}_d$, i. e., for $w < \bar{\bar{w}}$ where $\bar{\bar{w}}$ is implicitly defined by $\bar{\bar{N}}_s{}^0\,(\bar{\bar{w}}, M^n/p) = \tilde{N}_d{}^0\,(\bar{\bar{w}})$ with $\tilde{N}_d{}^0$ being given by (13). Thus $\bar{\bar{N}}_s$ cannot play a rôle that is completely analogous to that of unconstrained commodity demand, $\bar{Y}_d$, which is not subject to a similar constraint. Yet, we can easily define an *auxiliary labour supply* variable, $\bar{N}_s$, to do the job:

$$\bar{N}_s \equiv \begin{cases} \bar{\bar{N}}_s & \text{for } w \leq \bar{\bar{w}}, \\ \bar{\bar{N}}_s{}^0\left(\bar{\bar{w}}, \dfrac{M^n}{p}\right) + \dfrac{\partial \bar{\bar{N}}_s{}^0\,(\bar{\bar{w}}, M^n/p)}{\partial w}\,(w - \bar{\bar{w}}) & \text{for } w \geq \bar{\bar{w}}. \end{cases} \tag{43}$$

Fig. 2 shows that the curve depicting the influence of the wage rate on $\bar{N}_s$ looks rather familiar.

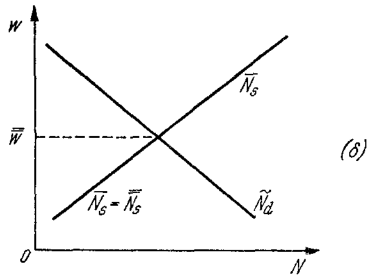

Fig. 2. The labour supply curve

Analogous to (38) and (39), $\bar{N}_s$ has appealing properties in a situation of temporary equilibrium:

$$\bar{N}_s \leq N_d \Rightarrow N_s = \bar{N}_s = N, \tag{44}$$

$$\bar{N}_s > N_d \Rightarrow N_s > N_d = N. \tag{45}$$

Thus, $\bar{N}_s$ is a quantitively correct indicator of N_s if employment is supply determined and a qualitatively correct indicator if it is demand determined. The proofs for (44) and (45) are given in the Appendix.

(44) and (45) ensure that for an analysis of temporary equilibrium the $\bar{N}_s$ curve can, in important respects, be used like a normal labour supply curve depicting a behavioural function. Note, however, that in fact it is not merely the graph of such a function. Instead it represents a whole menu of behavioural functions and equilibrium conditions. Note, furthermore, that the $\bar{N}_s$ curve is at

variance with the Keynesian labour supply curve since it refers to the real instead of the nominal wage rate and since its position will generally depend on the stock of real money balances.

f) Labour Demand and Commodity Supply

Because of their close relationship we discuss effective labour demand and effective commodity supply together. Combining (15) with (20) and (14) with (16) we have:

$$N_d = \min\,[\tilde{N}_d, \underset{(+)}{\Phi^{-1}}(Y_d)], \tag{46}$$

$$Y_s = \underset{(+)}{\Phi}\,[\min\,(\tilde{N}_d, N_s)]. \tag{47}$$

Since it was shown above that in many respects $\bar{Y}_d$ is able to represent Y_d and $\bar{N}_s$ is able to represent N_s, it is useful to consider the following *auxiliary variables*:

$$\bar{N}_d \equiv \min\,[\tilde{N}_d, \underset{(+)}{\Phi^{-1}}(\bar{Y}_d)], \tag{48}$$

$$\bar{Y}_s \equiv \underset{(+)}{\Phi}\,[\min\,(\tilde{N}_d, \bar{N}_s)]. \tag{49}$$

In the case of a temporary equilibrium they have the properties

$$\bar{N}_d \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \bar{N}_s \Rightarrow \bar{N}_d \left\{\begin{matrix} = \\ = \\ ? \end{matrix}\right\} N_d \left\{\begin{matrix} = \\ = \\ > \end{matrix}\right\} N \left\{\begin{matrix} < \\ = \\ = \end{matrix}\right\} N_s \left\{\begin{matrix} ? \\ = \\ = \end{matrix}\right\} \bar{N}_s \tag{50}$$

and

$$\bar{Y}_s \left\{\begin{matrix} < \\ = \\ > \end{matrix}\right\} \bar{Y}_d \Rightarrow \bar{Y}_s \left\{\begin{matrix} = \\ = \\ ? \end{matrix}\right\} Y_s \left\{\begin{matrix} = \\ = \\ > \end{matrix}\right\} Y \left\{\begin{matrix} < \\ = \\ = \end{matrix}\right\} Y_d \left\{\begin{matrix} ? \\ = \\ = \end{matrix}\right\} \bar{Y}_d. \tag{51}$$

The proofs for (50) and (51) can easily be given. From (38) [(44)] and (39) [(45)] we already know that (50) [(51)] is true in a temporary equilibrium if $\bar{N}_d = N_d$ [$\bar{Y}_s = Y_s$]. In the Appendix it is shown in addition that the only type of temporary equilibrium where this equation might not hold is characterized by $\bar{N}_d > N_s$ [$\bar{Y}_s > Y_d$] and $N_d > N_s$ [$Y_s > Y_d$]. Because of $N = \min\,(N_d, N_s)$ [$Y = \min\,(Y_d, Y_s)$] we obviously have $\bar{N}_d > N_s = N$ [$\bar{Y}_s > Y_d = Y$] in this case, and furthermore, by definition of $\bar{N}_s$ [$\bar{Y}_d$], $\bar{N}_d > \bar{N}_s = N_s = N$ [$\bar{Y}_s > \bar{Y}_d = Y_d = Y$]. Thus, properties (50) and (51) are seen to hold throughout in a temporary equilibrium.

These properties ensure that $N = \min\,(\bar{N}_d, \bar{N}_s)$, $Y = \min\,(\bar{Y}_d, \bar{Y}_s)$, $\operatorname{sgn}(\bar{N}_d - \bar{N}_s) = \operatorname{sgn}(N_d - N_s)$, and $\operatorname{sgn}(\bar{Y}_d - \bar{Y}_s) = \operatorname{sgn}(Y_d - Y_s)$.

Thus any symbol with an upper bar is able to represent the corresponding symbol without a bar, provided that it is sufficient to have quantitative information about the trading volumes in the labour and commodity markets, but only qualitative information about excess demands and supplies. For the present analysis this condition is satisfied. In verbal discussions, we shall henceforth no longer distinguish between the two types of variables.

It will be helpful to have a brief look of Figs. 3 and 4 which illustrate the derivation of $\bar{N}_d$ and $\bar{Y}_s$ as given by (48) and (49). Both figures consist of diagrams β, γ, and δ. Diagram β, which is also contained in Fig. 1, generally shows the aggregate commodity demand and supply curves as represented by $\bar{Y}_d$ and $\bar{Y}_s$, diagram γ the graph of the production function, and diagram δ, from Fig. 2, the labour demand and supply curves as given by $\bar{N}_d$ or $\tilde{N}_d$, respectively, and $\bar{N}_s$. As a starting point assume first that firms can sell the output and buy the labour they want under all possible wage rate and commodity price combinations. Then effective labour demand and commodity supply are equal to their notional counterparts and thus determined by the real wage rate alone. This can be seen in Fig. 4 if we consider only the curves $\tilde{N}_d$ and $\Phi(N)$. Given the wage rate OA, the firms' labour demand is OC and according to the production function their commodity supply is OD, independent of the price level. The question is now the way in which quantity constraints that the firm might face will modify this result.

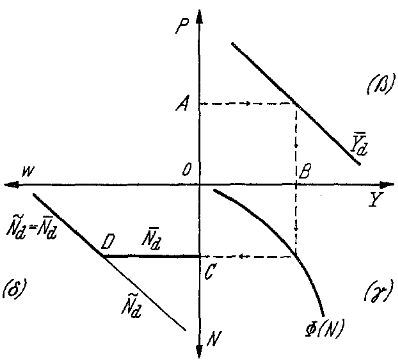

Fig. 3. The labour demand curve

Fig. 3 illustrates how, according to (48), an output constraint affects the labour demand curve. Given the price level OA and the commodity demand curve $\bar{Y}_d$, firms cannot sell more than OB, and thus their maximum labour demand is OC. This implies that the effective labour demand curve $\bar{N}_d$ has the horizontal branch DC and coincides with the notional demand curve $\tilde{N}_d$ to the left of and above D.

Fig. 4 illustrates how, according to (49), an employment constraint affects the commodity supply curve $\bar{Y}_s$. Given the real wage rate OB and the labour supply curve $\bar{N}_s$, firms cannot — although they would like to — get more labour than OC. Thus, their effective commodity supply is OD. Since labour supply is a function of the real wage rate, but not of the commodity price, this implies a

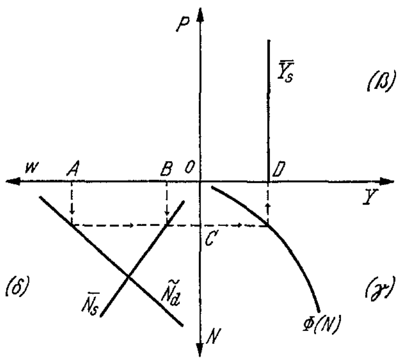

Fig. 4. The commodity supply curve

vertical commodity supply curve $\bar{Y}_s$. As explained above, the same curve is obtained if the wage rate is OA so that firms choose the employment level OC deliberately.

5. The Working of the Model Under Exogenous Parameter Changes

The working of the model can be studied systematically if we classify the possible types of temporary equilibria according to the states of the commodity and labour markets (see Malinvaud, 1977, p. 31)[28] (Table 2).

Table 2 illustrates ten different cases. However, as a comparison between (46) and (47) shows, three of these, (d), (h), and (i), cannot occur. The reason is that factories will signal excess supply in the commodity market only if they still enjoy some free play in the

[28] Malinvaud considers only cases (a), (c), (e), and (j) and calls case (c) 'classical unemployment'.

labour market and will express excess demand in the labour market only if they believe that they can sell more commodities than they actually do. Note, however, that independently of a constraint on its labour demand, a firm might well face a constraint on its investment demand. Thus cases (g) and (j) are clearly possible.

Table 2. Possible Market Situations, Classified by Effective Demand and Supply

Labour market	Commodity market		
	Excess supply	Cleared	Excess demand
Excess supply	Keynesian unemployment (a)	Classical unemployment (b)	Stagflation (c)
Cleared	— (d)	Walrasian equilibrium (e)	Demand inflation (g)
		Administered underemployment (f)	
Excess demand	— (h)	— (i)	Repressed inflation (j)

We want to find out what these and the other possible cases of the above classification scheme imply for the variables of the model and under which circumstances they become relevant. For this purpose we examine the performance of the model under various exogenous shocks assuming that the economy is initially in a state of *Walrasian equilibrium*, defined such that no agent, except traders[29], faces a strictly binding constraint, i. e., $\overline{Y}_d=\overline{Y}_s$, $\overline{N}_d=\overline{N}_s$, $\overline{N}_d=\tilde{N}_d$, $\overline{Y}_s=\Phi(\tilde{N}_d)$.

In principle we can still carry out the analysis in a comparative static framework, observing the economy's instantaneous reactions to shocks. However, we shall also add some dynamic considerations without claiming to give a comprehensive analysis.

With the passage of time and the corresponding observation of new market signals all agents continuously reconsider their decisions. They shift the 'present' period forward, with its length unchanged,

[29] Of course, this exception implies that we have a Walrasian equilibrium in a restricted sense only. The assumption of traders facing constraints in all market situations could be removed only by abolishing the assumption of increasing returns to scale for the transactions technology which, however, is firmly grounded in the Baumol-Tobin theory.

let the 'future' begin thereafter, and resolve the optimization problems of section 3[30]. For each point in time the economy again is in a temporary equilibrium of the kind discussed above.

Generally, the temporary equilibrium alters with time since the variables that were treated above as exogenous are now gradually changing, with the change being determined by the kind of temporary equilibrium currently existing. Thus, the stock of nominal money balances increases if government expenditure is positive, the capital stock changes if investments are different from zero, prices are subject to change in the case of excess demand or supply, and there will be changes in the expectational patterns of market agents, the determinants of which are, however, not well known. In this paper we restrict our attention to the rôle of price changes[31]. It is assumed that, due to a high speed of these changes, the period of time required for the adjustment processes to be discussed is very short: short enough to render negligible endogenous stock adjustments in capital and nominal money balances. In addition, the expectational hypotheses summarized in Table 1 are maintained with exogenous variables being time-invariant.

The price adjustment hypothesis we use is

$$\operatorname{sgn} \dot{p} = \operatorname{sgn} (\bar{Y}_d - \bar{Y}_s), \tag{52}$$

where the dot indicates a time derivative. We alternately consider the cases where the real wage rate is constant over time due to an appropriate trade union policy and where it adjusts to clear the labour market. For reasons that are explained below we do not introduce an explicit formal assumption about the wage adjustment process similar to (52). To simplify the discussion the trade margin is kept constant throughout. Furthermore the influence of real money balances on labour supply is disregarded since the direction of this influence is ambiguous [cf. (41)].

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[30] This specification has been borrowed form Barro/Grossman (1976). It seems to be a fairly realistic description of the way agents plan, but it also has its drawbacks. It implies that with the passage of time the behaviour derived from repetitive planning does not coincide with the behaviour derived from the original plan, even if the market data develop in the way which was assumed under the original plan. This implication is not in line with the Bellman principle for rational intertemporal planning.

[31] Other studies in price dynamics are, e. g., provided by Barro/Grossman (1978), Benassy (1978) and Böhm (1980, pp. 110—113). A discussion of the dynamics arising from an imbalanced government budget can be found in Böhm (1978 a, 1978 b, 1980, pp. 102—109).

a) Keynesian Unemployment, Administered Underemployment, and Walrasian Equilibrium

The model developed above is represented graphically in Fig. 5. This figure combines Figs. 1—4 and consists of the diagrams α—δ which have been explained before. The points labeled (1) indicate the economy's initial situation at Walrasian equilibrium.

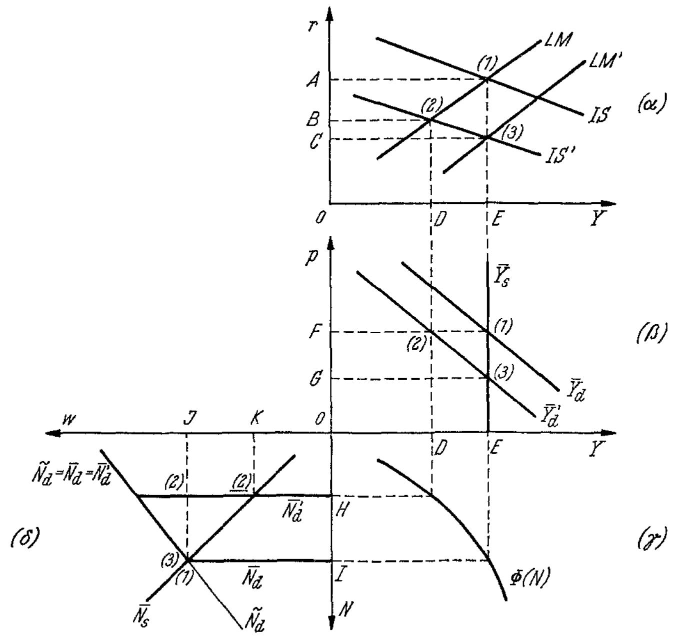

Fig. 5. An exogenous decrease in commodity demand

Suppose that in this situation an exogenous reduction in demand occurs[32]. Then the *IS* and the commodity demand curves shift to the left to *IS'* and $\bar{Y}_d'$. Since the commodity price is sticky, this imposes a restriction on factory output which shifts the horizontal branch of the effective labour demand curve upward to $\bar{N}_d'$. Given the commodity price and the real wage rate the new position of the economy is indicated by points (2). It is characterized by a fall in the interest rate by *AB*, an excess supply in the commodity market as indicated by *DE*, and involuntary unemployment (excess

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[32] Among the possibilities are a reduction in government spending and a reduction of investment demand for each given rate of interest due to a change in the factory's expectations of future market data.

supply) in the labour market as indicated by *HI*. Obviously this situation characterizes *Keynesian unemployment*, case (a) in the above classification scheme.

Since it was a decrease in commodity demand which produced Keynesian unemployment it is possible for the government to cure the situation by increasing its expenditure and/or by stimulating private expenditure through a stock transfer of money balances[33]. This would shift the *IS* curve and/or the *LM* curve to the right; in any case it would be possible to shift the commodity demand curve back to its original position, so that the economy is restored to *Walrasian equilibrium.*

Even if the government does not increase the level of demand, the economy will return to Walrasian equilibrium by itself. If the real wage rate is fixed at its Walrasian level despite the involuntary unemployment, there will be a tendency for the commodity price to fall as long as there is excess supply in the commodity market. The decline in the price level raises the real stock of money, so that the *LM* curve shifts to the right to its new position *LM'*, where it intersects the *IS'* curve at $Y = \bar{Y}_s$. In diagram β this shift implies a movement along the aggregate demand curve down to the intersection point with the aggregate supply curve. This in turn ensures that the horizontal branch of the effective labour demand curve in diagram δ shifts back to its initial position $\bar{N}_d$. The new situation of the economy, labeled (3), is again a *Walrasian equilibrium.* Compared with the initial Walrasian equilibrium (1) it is characterized by a fall in the price level of *GF* and a fall in the rate of interest of *AC*.

The automatic stabilization mechanism just described works under the assumption of an exogenously given real wage rate. An

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[33] There are three channels through which the corresponding change in the rate of interest affects aggregate demand: 1. Investments rise. 2. Consumption may change due to the usual income and substitution effects. 3. Transaction costs fall since more money is used. The latter has a direct and indirect effect. The direct effect is a decrease of commodity demand for transaction purposes. The indirect effect is an increase in private consumption due to a reduction in the transaction cost. This third channel of influence represents the real balance effect. Interestingly enough, with the present specification, this effect is not necessarily positive. On the contrary, if the marginal rate of consumption is less than unity and if the household utility function is separable with respect to current employment, then the effect will be negative, for it works like a negative Haavelmo effect. To establish a positive real balance effect one has to assume that non-market resources are required to manage the transaction process.

interesting question is what would happen in the model if we assumed a competitive wage rate formation process, one of the possible ways to make the wage rate endogenous. In a competitive labour market involuntary unemployment would naturally induce a fall in the real wage rate. According to Fig. 5 this fall implies a shift to the left of the aggregate supply curve and, consequently, the price level does not have to decrease as much as before in order to clear both markets. Thus it seems that the economy could end up in a situation anywhere between points (2) and (3) on the commodity demand curve and points (1) and (2) on the labour supply curve when both markets clear. A very drastic example would be the limiting case of an instantaneous fall in the real wage rate from *OJ* to *OK*. This fall would obviously clear the labour market and, since it implies a shift to the left of the commodity supply curve until this curve intersects the demand curve at point (2), it would also clear the commodity market.

One might be tempted to suppose that in this situation there is no need for a further adjustment of market prices, so that the economy gets stuck in a situation of voluntary underemployment. Indeed, as shown by Benassy (1978, pp. 530 ff.), this happens if we supplement our assumption (52) about the commodity price adjustment mechanism by a similar assumption about the wage rate adjustment mechanism[34]. Yet, this result would by no means be plausible for a competitive labour market. In the current situation it would always pay instead for a single firm to increase the wage rate and lower the output price a little, since this firm could then get as many workers and sell as much output as it wanted. Of course, since all firms are assumed to behave in the same way, they can only partially remove their restrictions with a small price change. However, the incentive for further adjustments in both prices will persist. So, step by step, the economy will eventually reach *Walrasian equilibrium*. Only at that stage is there no further incentive for a change in prices.

Thus there appears to be no real danger that the competitive economy would remain in an underemployment situation[35]. But

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[34] Barro/Grossman (1976, pp. 95—99) contend that the economy will always return to Walrasian equilibrium after an initial disturbance. This, however, does not follow from their price formation equations (2.29) and (2.30).

[35] This statement is of course conditional on the framework of our model. Cf., e. g., Negishi (1978) who shows in the context of a risk theoretic model of labour supply determination that a wage rate *above* the one which clears the labour market might persist.

what would happen if there were, for example, a government controlled wage cut "in order to induce firms to employ more workers"? In this case firms must accept a real wage rate below the Walrasian level as a datum and thus there would be no way to shift the aggregate commodity supply curve back to the Walrasian position. The economy would be in *administered underemployment* with all markets cleared; case (f) in our classification scheme.

b) Demand Inflation and Walrasian Equilibrium

The effect of an exogenous increase in demand is illustrated in Fig. 6, which is similar to Fig. 5. As before, the economy is initially in Walrasian equilibrium, indicated by (1). After the increase in demand the *IS* curve and the commodity demand curve shift to the

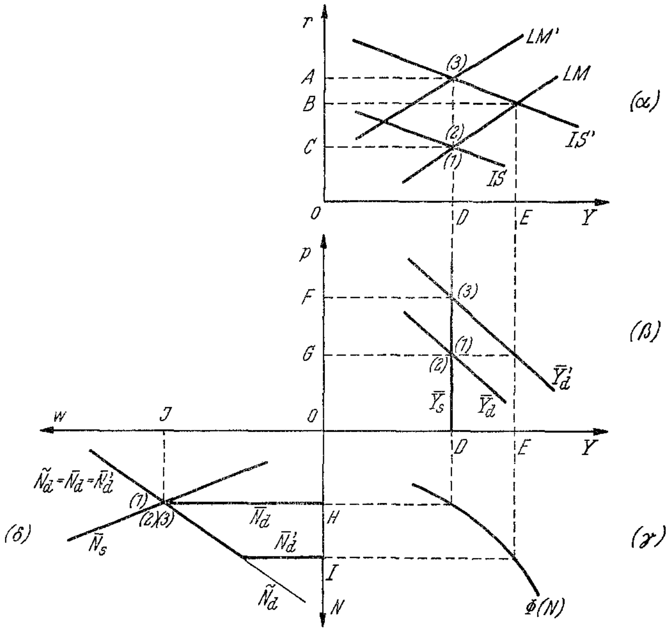

Fig. 6. An exogenous increase in commodity demand

right to their new positions *IS'* and $\bar{Y}_d'$. For a given commodity price this implies that the effective labour demand curve moves from $\bar{N}_d$ to $\bar{N}_d'$.

The new short-run position of the economy is at (2). Obviously very little has happened in the system: Employment, production, the real wage rate, the commodity price, and the rate of interest are all unchanged. [For the latter cf. Fig. 1]. Only commodity demand has risen as indicated by *DE*. The state of the economy is described by case (f) in the above classification scheme and is obviously such that an inflationary process must start. Referring to its cause, we call it *demand inflation.*

The inflationary process decreases the real stock of money and therefore shifts the *LM* curve to the left, reducing commodity demand. The reduction of commodity demand is represented in diagram β by an upward movement along the new commodity demand curve $\overline{Y}_d'$, inducing an upward shift of the horizontal branch of the effective labour demand curve in diagram δ. The whole process comes to a halt when the *LM* curve has attained the position *LM'*, and the effective labour demand curve is back in the original position $\overline{N}_d$. The new situation, (3), is again a *Walrasian equilibrium.* The only difference from the initial equilibrium is that the rate of interest has risen by *CA* and the commodity price by *GF.*

Although the real wage rate has implicitly been assumed constant in these considerations, nothing substantial would alter if we make it endogenous by assuming a competitive labour market. Since the intersection point of the labour supply curve and the effective labour demand curve does not move during the whole adjustment process, the real wage rate which clears the labour market would not change.

c) Repressed Inflation and Administered Underemployment

To assume a competitive labour market certainly presents an extremely idealized view of reality. Both collective bargaining and frequent government intervention in the labour market in reality quite often bring about a real wage rate which deviates substantially from its market clearing level. Therefore it is interesting to check which adjustments would take place in our model if there were an exogenous change in the real wage rate which has to be accepted by both the representative firm and the representative worker.

We start with a decrease in the real wage rate, assuming again that initially the economy is (now more or less by chance) in a Walrasian equilibrium. The outcome of this change is illustrated in Fig. 7, in which the starting position of the economy is again labeled (1). Suppose that in diagram δ the wage rate drops from *OJ* to *OK*. Then labour supply falls to *OH* and restricts firms, so that their commodity supply curve shifts to the left from $\bar{Y}_s$ to $\bar{Y}_s'$ in diagram β. The output level is now supply determined and lower than before by DE. Due to the lower transactions volume the demand for money declines and makes the rate of interest fall from

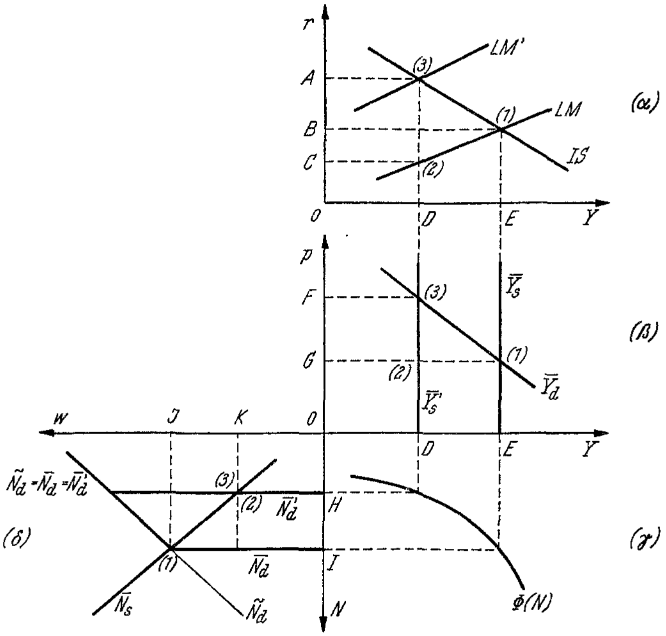

Fig. 7. An exogenous decrease in the real wage rate

OB to OC as shown in diagram α. The new short-run position of the economy is indicated by (2). It is characterized by an excess demand in both the labour and the commodity markets, case (j) in the introductory classification scheme. Since an inflationary process is now initiated in the output market and since the reason for this process is a reduction in labour supply, this case is called *repressed inflation.*

The adjustments occurring in the model due to the inflationary process are similar to the case of demand inflation discussed above. The LM curve shifts to LM' since the real stock of money is declining, aggregate demand goes down (represented by an upward movement along the $\bar{Y}_d$ curve), and the horizontal branch of the effective labour demand curve shifts upward until the demand curve reaches its final position $\bar{N}_d'$. The final situation of the economy is indicated by (3). Compared with the initial situation the price level and the interest rate have risen by *GF* and *BA* respectively, and output and employment have declined by *DE* and *HI*. The state of the economy is now the same as under a wage cut in a situation of Keynesian unemployment, as discussed above. Provided that the real wage rate stays fixed below its Walrasian level an *administered underemployment* persists with all markets cleared; case (f) in the classification scheme.

d) Stagflation and Classical Unemployment

The case of an exogenous increase in the real wage rate is illustrated in Fig. 8. Let us assume that, starting with the original Walrasian equilibrium (1), the real wage rate rises from *OL* to *OK*

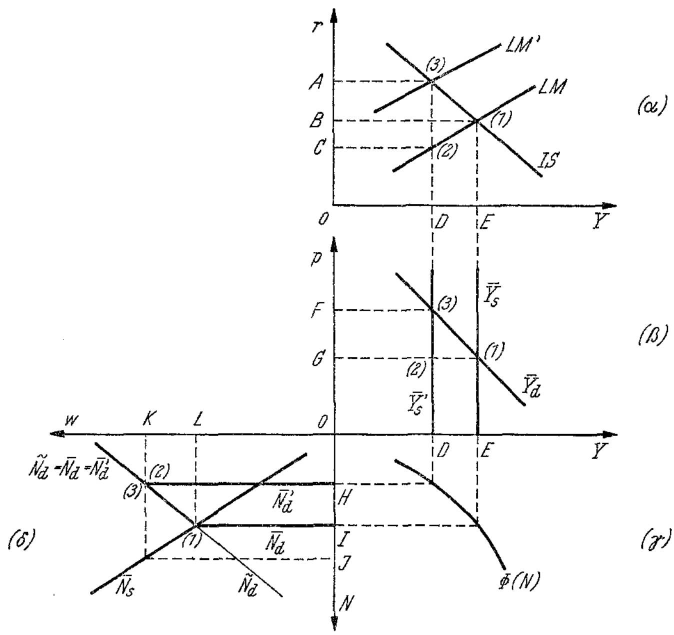

Fig. 8. An exogenous increase in the real wage rate

in diagram δ. According to the effective labour demand curve $\bar{N}_d$, this implies a reduction in labour demand and employment by *HI*. Corresponding to this reduction is a shift to the left of the commodity supply curve in diagram β from $\bar{Y}_s$ to $\bar{Y}_s'$ which reduces the production volume by the same amount *DE*. The decrease in the production volume lowers the rate of interest by the amount *BC*, illustrated in diagram α. The short-run position (2) which the economy will attain after these adjustments is characterized by excess demand (*DE*) in the commodity market and excess supply (*HJ*) in the labour market. Since on the one hand excess demand in the output market induces a price level increase and since on the other hand excess supply in the labour market is involuntary unemployment, this situation is usually called *stagflation*; case (c) in the introductory classification scheme.

The increase in the price level now induces the same kind of adjustment process as we described for previous cases. The *LM* curve shifts to *LM'*, commodity demand falls to *OD*, and the horizontal branch of the effective labour demand curve shifts up to $\overline{N}_d'$. In comparison with the initial situation of the economy (1), the final position (3) is characterized by a reduction in output and employment by *DE* and *HI* respectively and by the creation of involuntary unemployment as indicated by *HJ*. Since the commodity market has eventually cleared we are in a situation which is characterized by case (b) in the above classification scheme. It is denoted *classical unemployment* since it seems to characterize the pre-Keynesian concept of unemployment. In this case a wage cut would, as classical economists suggested, indeed be the best remedy for curing the economy's disease.

6. Concluding Remarks

In the present paper we derive a temporary-equilibrium model from the behaviour of individual maximizers. The most important aspect of the model is the rehabilitation of the Keynesian consumption function and a simple labour supply function. However, in addition it also incorporates the usual transactions demand for money function and investment demand function. Armed with these familiar tools we could explain rather easily various types of disequilibrium situations.

Despite these attractive aspects of the model there are some serious shortcomings; for example, we have ignored taxation, inflationary expectations, and stock adjustments. These omissions may be incorporated into a fully integrated model.

The major problem in models of the Clower/Barro/Grossman variety is the imposition of rationing schemes. We adopted usual assumptions and, with the 'investment last' principle, we introduced a particular rationing scheme in order to approximate the buffer stock function of inventory capital. Bur clearly we need a theory instead of assumptions. Such a theory would also have to explain the price setting behaviour rather then exogenously imposing this behaviour upon the firms.

Appendix

Ad (38). Suppose there is a temporary equilibrium where $\bar{Y}_d \leq Y_s$ but $Y_d \neq Y$. Then $Y = \min(Y_s, Y_d)$ implies $Y = Y_s < Y_d$. Because of $\partial C_d^2/\partial Y + \partial H_d^1/\partial Y < 1$ and (31), $Y < Y_d$ requires that the situation of the economy is represented by a point (r, Y) on the *LM* curve left of the *IS* curve. Since the *LM* curve is positively sloped this point must also be left of the intersection of the two curves, i. e., $Y < \bar{Y}_d$. Together with $\bar{Y}_d \leq Y_s$ the latter implies $Y < Y_s$, which contradicts $Y = Y_s$. Thus we find $Y_d = Y$ and by definition of $\bar{Y}_d$, $Y_d = Y = \bar{Y}_d$, Q. E. D.

Ad (39). Because of $Y = \min(Y_d, Y_s)$ the inequality $\bar{Y}_d > Y_s$ implies $\bar{Y}_d > Y$, i. e., the temporary equilibrium is characterized by a point (r, Y) on the *LM* curve left of its intersection with the *IS* curve. Due to the slopes of these curves and because $\partial C_d^2/\partial Y + \partial H_d^1/\partial Y < 1$ this implies $Y_d > Y$. Since $Y = \min(Y_d, Y_s)$ the latter ensures $Y_s = Y$, Q. E. D.

Ad (44). Suppose there is a temporary equilibrium where $\bar{N}_s \leq N_d$, yet $N_s \neq N$. Then $N = \min(N_s, N_d)$ implies $N = N_d < N_s$. Since in the situation at hand $w \leq \bar{\bar{w}}$ and $\bar{N}_s = \bar{\bar{N}}_s$, (42b) ensures that $N_s > N$ indicates a point (w, N) left of the $\bar{\bar{N}}_s$ curve, i. e., $N < \bar{\bar{N}}_s = \bar{N}_s$. Obviously the latter contradicts $\bar{N}_s \leq N_d = N$. Thus $N_s = N$, and by definition of $\bar{N}_d$, $N_s = \bar{N}_s = N$, Q. E. D.

Ad (45). Since $N = \min(N_s, N_d)$ and $N_d \leq \tilde{N}_d$ the inequality $\bar{N}_s > N_d$ implies that the situation of the economy is now characterized by a point (w, N) on the $\tilde{N}_d$ curve or left of it, which, figuratively speaking, can be reached from a point on the $\bar{\bar{N}}_s$ curve by moving left and/or upwards. By the definition of $\bar{\bar{N}}_s$ and because of (42a) and (42b) this implies $N_s > N$. Because of $N = \min(N_s, N_d)$ the latter in turn gives $N = N_d$, Q. E. D.

Ad (50). It is shown that a temporary equilibrium with $\bar{N}_d \neq N_d$ requires $N_d > N_s$ and $\bar{N}_d > N_s$. (a) Suppose $\bar{Y}_d \leq Y_s$. Then (38) implies $\bar{Y}_d = Y_d$ so that according to (46) and (48) $N_d = \bar{N}_d$. (b) Suppose $\bar{Y}_d > Y_s$ so that according to (39) $Y_d > Y_s$. Because of (47) this case is characterized by $\Phi^{-1}(\bar{Y}_d) > \min(\tilde{N}_d, N_s)$ and $\Phi^{-1}(Y_d) > \min(\tilde{N}_d, N_s)$. (ba) Consider the subcase where $\tilde{N}_d \leq N_s$. Here (46) and (48) imply $\tilde{N}_d = N_d = \bar{N}_d$. (bb) Consider now the other subcase where $\tilde{N}_d > N_s$ and consequently $\Phi^{-1}(\bar{Y}_d) > N_s$ as well as $\Phi^{-1}(Y_d) > N_s$. From (46) and (48) we see that this case must be characterized by both $N_d > N_s$ and $\bar{N}_d > N_s$, Q. E. D.

Ad (51). It is shown that a temporary equilibrium with $\bar{Y}_s \neq Y_s$ requires $Y_s > Y_d$ and $\bar{Y}_s > Y_d$. (a) Suppose $\bar{N}_s \leq N_d$. Then (44) implies $\bar{N}_s = N_s$ so that according to (47) and (49) $Y_s = \bar{Y}_s$. (b) Suppose $\bar{N}_s > N_d$ so that according to (45) $N_s > N_d$. Because of (46) this case is characterized by $\bar{N}_s > \min[\tilde{N}_d, \Phi^{-1}(Y_d)]$ and $N_s > \min[\tilde{N}_d, \Phi^{-1}(Y_d)]$. (ba) Consider the subcase where $\tilde{N}_d \leq \Phi^{-1}(Y_d)$. Here (47) and (49) imply $\Phi(\tilde{N}_d) = Y_s = \bar{Y}_s$. (bb) Consider now the other subcase where $\tilde{N}_d > \Phi^{-1}(Y_d)$ and consequently $\bar{N}_s > \Phi^{-1}(Y_d)$ as well as $N_s > \Phi^{-1}(Y_d)$. From (47) and (49) we see that this subcase must be characterized by both $Y_s > Y_d$ and $\bar{Y}_s > Y_d$, Q. E. D.

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