

The Fossil Episode

John Hassler
Hans-Werner Sinn

CESIFO WORKING PAPER NO. 4016
CATEGORY 10: ENERGY AND CLIMATE ECONOMICS
DECEMBER 2012

An electronic version of the paper may be downloaded

- from the SSRN website: www.SSRN.com
- from the RePEc website: www.RePEc.org
- from the CESifo website: www.CESifo-group.org/wp

The Fossil Episode

Abstract

We build a two-sector dynamic general equilibrium model with one-sided substitutability between fossil carbon and biocarbon. One shock only, the discovery of the technology to use fossil fuels, leads to a transition from an initial pre-industrial phase to three following phases: a pure fossil carbon phase, a mixed fossil and biocarbon phase and an absorbing biocarbon phase. The increased competition for biocarbon during phase 3 and 4 leads to increasing food prices. We provide closed form expressions for this price increase. Our calibration leads to a price increase of 40% if capital and labor are allowed to move to the biocarbon sector. Otherwise, the price increase is much higher. We also use the model to analyze the consequences of restrictions on using biocarbon as fuel. We show that such restrictions can lead to a substantially slower global warming due to an endogenous slowdown of fossil fuel extraction.

JEL-Code: Q540, Q320, O410.

Keywords: hotelling rule, fossil carbon, biocarbon, transition phases, oil price, food price, global warming.

John Hassler
IIES
Stockholm University
10691 Stockholm
Sweden
john@hassler.se

Hans-Werner Sinn
Ifo Institute – Leibniz-Institute for
Economic Research at the
University of Munich
Poschingerstraße 5
81679 Munich
Germany
sinn@ifo.de

This version: Dec 4, 2012.

We are grateful for comments from Per Krusell and Rick van der Ploeg as well as from seminar participants in Madrid, Sigtuna and Stockholm. We thank Wolfgang Meister and Julio Saavedra for excellent editorial assistance.

1 Introduction

The price of a barrel of oil was US\$99 in January 2012. Using an energy content of 1588kWh/barrel, this gives a price per kWh of 6.2 cents. In the same month, the world market price of corn was US\$273 per ton. Using an energy content of 4389kWh per ton, the price per kWh in the form of corn is also 6.2 cents.¹ In Figure 1, we plot yearly prices of oil and corn in a long historic perspective, both expressed per unit of energy content. As we can see, the price of oil has converged slowly to the price of corn from below, reaching it in the last decade and coinciding with it rather closely since 2005.

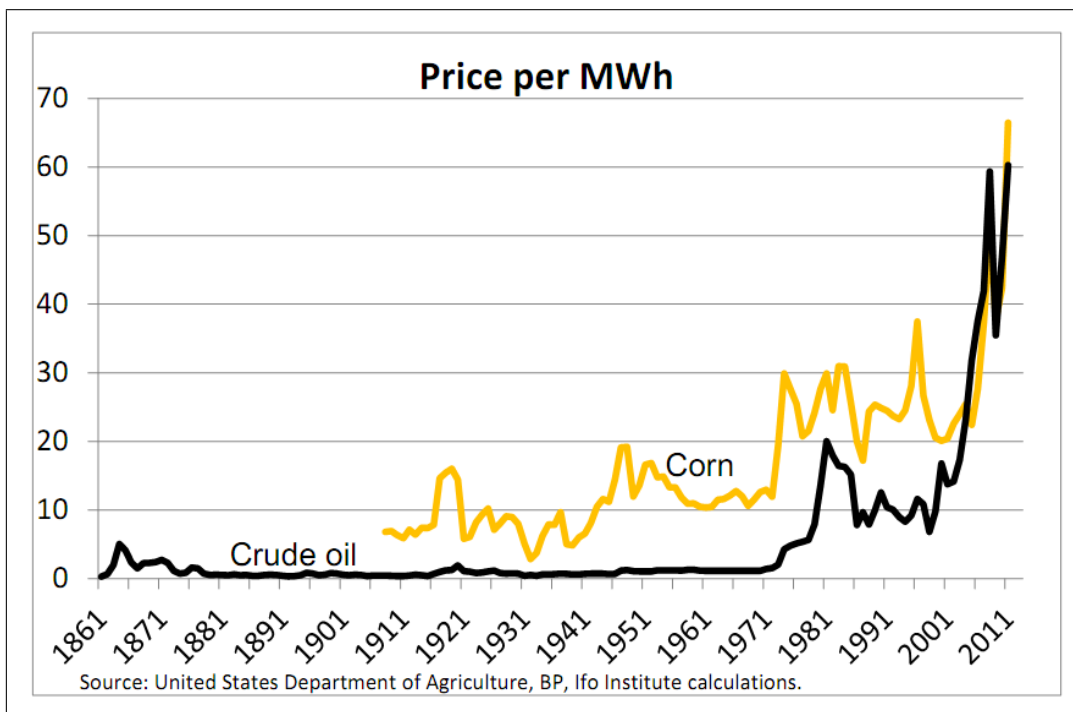


Figure 1. Price in US\$ per MWh. Corn and Crude Oil.

Since the point in time when the two prices first were equal, the two prices have been tracking each other in a way that has not occurred before. The closest parallel before that point occurred during the oil crisis in the 1970's and early 80's, when the price of oil almost reached the price of corn and there also was a tendency for comovements.

Figure 2 depicts the correlation between monthly oil and corn prices over a rolling 10-year backward-looking window. As we can see, the correlation between oil and corn prices have been very high for the last 10 years or so. A period of high correlation also occurred

¹The energy content of corn is taken from Penn State College of Agricultural Sciences, (2012) as the energy content of shelled corn taking into account a water content of 15%. The energy content of oil is from EIA (2012).

during the oil crisis when the correlation between the prices was high, albeit not as high as recently. The correlation between the series during months when the price of corn is less than 120% of the oil price is 0.89, while it is only 0.45 for the other months. In growth rates, the corresponding correlations are 0.32 and -0.06.

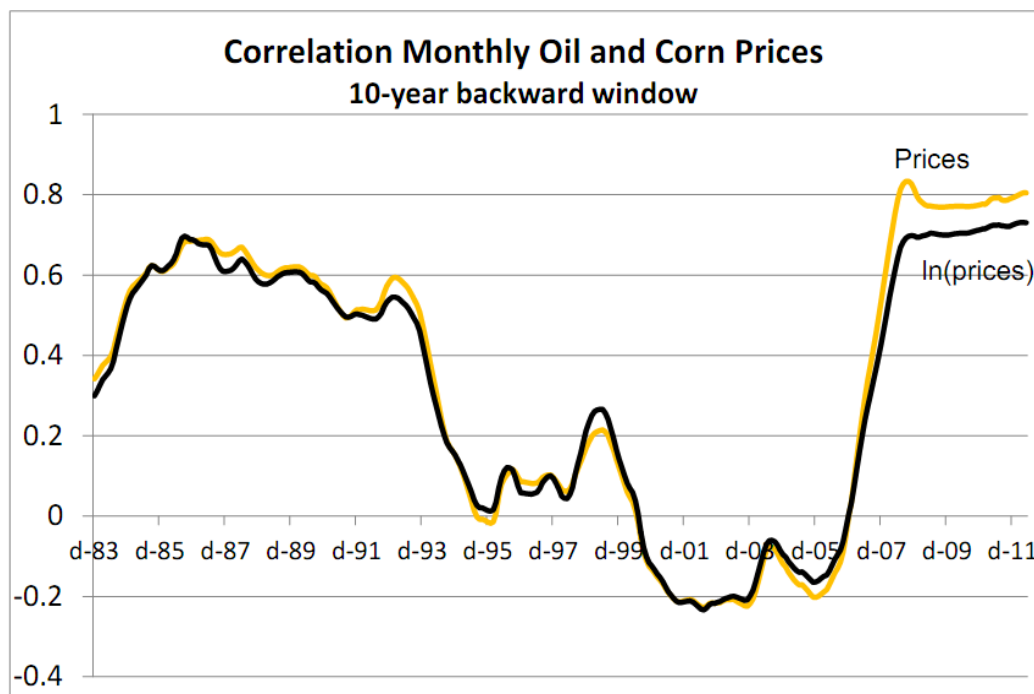


Figure 2. Correlation between monthly oil and corn prices. 10 year backward moving window.

The fact that energy prices have reached the level of food prices when expressed in terms of energy content may have profound consequences for the economy. In many applications biocarbon is an almost perfect substitute for fossil carbon. In fact, in 2007, 8% of US agricultural land, or 30% of US total corn output, was used for the production of bioethanol,² and in the fiscal year 2010/2011, nearly 40% of the US corn harvest was used for this purpose.³ Corn can easily be used as fuel for heat production, both industrially and in owner-occupied housing.

The starting point of this paper is that we now may be witnessing a transition to a new phase in economic history. For two hundred years, since the start of the Industrial Revolution when mankind learned to tap fossil fuels as a resource, fossil carbon has been a much cheaper source of energy than biocarbon. Its abundance may well have liberated about half of the

²von Braun (2008).

³National Corn Growers Association (2008, 2012).

land available from the production of fodder for draught and pack animals,⁴ and made it possible to enhance both general standards of nutrition and the energy input to industrial production, fueling an unprecedented growth process. However, over time, the exhaustible fossil fuel resource became scarcer, and the fossil fuel price continued to rise relative to the food price, eventually reaching that price, making it more attractive for farmers to deliver their crops to the filling station instead of the grocery store.

We have already seen indications of the increased competition between different uses of biocarbon. In January 2007, thousands of people protested against rising food prices in Mexico City, an event known as the Tortilla crisis. Corn, imported from the US, had become nearly twice as expensive as during the previous winter, increasing the price of tortillas by 35%. A year later, wheat and rice prices had trebled, and the hunger riots had spread to 37 countries.⁵ Arguably, this development was caused by the rapid increase in the production of bioethanol from corn in the US, and to a lesser extent by the rise of biodiesel in Europe and bioethanol in Latin America. While ethanol made from corn played hardly any role until the late 1970s, it became a dominant economic factor in US farming in the first decade of this century. By the time of the Tortilla crisis, almost the entire increase in the world's corn production from 2004 until 2007 ended up as input for the ballooning US bioethanol production.⁶

According to a study by the International Food Policy Research Institute (IFPRI), biofuel production accounted for 30% of the rise in the average price of corn, rice and wheat between 2000 and 2007,⁷ and according to Mitchell (2008), most of the price increase resulted from soaring oil prices. These findings triggered a heated debate. Authors like Piesse and Thirtle (2009), Headey and Fan (2008) or Collins (2008) basically supported Mitchell. Gilbert (2010) and Ajanovic (2010) were more critical about causality, but even they did not deny the close substitutability of food and energy.

While it is difficult to make long-run predictions regarding the development of new technologies for energy production, it seems likely that the competition for the use of biocarbon will increase. Increased scarcity of fossil fuels is one driving force for this. Another is that biocarbon is an alternative to fossil energy that avoids the greenhouse effect. Biocarbon has a big advantage over many other green energy sources insofar as it can easily be turned into a liquid fuel that is a nearly perfect substitute for gasoline and diesel fuels.

Although the discussion about the causes of the Tortilla crisis is not yet fully settled, it

⁴See Sinn (2012, p. 117)

⁵See Sinn (2012, chapter 3, "Table or Tank").

⁶World corn production rose during this period by 55 million tons, while the US consumption of corn for bioethanol production rose by 50 million tons. See Mitchell (2008).

⁷See Rosegrant (2008).

is a fact that food and fossil fuel prices have converged and have shown a close comovement in recent years. It is also a fact that this has triggered an increase in the use of biocarbon as fuel. We are therefore inclined to see the Tortilla crisis as a turning point in history at which the fossil energy price reached the food price and led to a sudden linkage between the food and energy markets.

This sudden linkage likely resulted from the one-way substitutability between fossil carbon and biocarbon. Biocarbon can be used as energy for technical uses, but fossil fuel cannot be consumed as food. All attempts to chemically convert the carbon hydrogen chains in fossil carbon fuel into edible substances have failed. This one-way substitutability implies that food and energy markets were separate when the fossil energy price was below the food price, both prices being expressed in terms of energy content, and coupled all of a sudden when the fossil energy price reached the food price. Sinn (2012) called this phenomenon "ratchet effect".

The purpose of this paper is to analyze the consequences of the one-way substitutability between fossil carbon and biocarbon using a global growth model. We develop a long-term intertemporal dynamic general equilibrium model with an industrial sector operating with capital, labor and energy as inputs, and an agricultural sector using land, capital and labor. With just one unforeseen shock, the appearance of a technical device to exploit the fossil fuel reservoirs, namely steam and combustion engines (the Industrial Revolution), we can endogenously derive a development path with four stages: 1. A pre-industrial stage where food and fodder rival for land, and bioenergy is the only form of energy available. 2. A fossil phase where land is exclusively used for the production of food while energy is taken exclusively from fossil sources. 3. A mixed phase where biocarbon and fossil carbon are both used as a source of energy and an increasing share of land is gradually used for biocarbon production. 4. A final biophase where the stock of fossil carbon is exhausted and, as in the pre-industrial phase, biocarbon serves both nutrition and energy needs.

We use the model to illustrate how food prices would be expected to develop over time. A key result is that if capital and labor are free to move to the biocarbon producing sector, the increase in the food price caused by the transition to a fossil free economy need not be very large. Under a benchmark calibration, the price increases by about 40% during the transition. However, the price increase may be much larger if the output expansion of the biocarbon sector is hindered while the use of biocarbon as fuel is not restricted. In the extreme, if the distribution of capital and labor over the production sectors is fixed, the price increase during the transition is a whopping 250%. We perform a comparative statics exercise showing, in particular, that the smaller the share of the household budget used for food purchases is, the larger the implied price increase.

We also append the growth model with a simple climate module and a carbon circulation system. We do this in order to examine the claim that public policies stimulating the use of biocarbon would mitigate the global warming problem. Here we show that the reverse is true, a ban or restrictions on biocarbon use for fuel use can substantially slow down global warming. In addition, it would decouple food and fuel prices, at least for an extended period of time.

The model we develop can be seen as a quite natural extension of standard growth models with energy following Dasgupta and Heal (1974). Our model is tractable and transparent and provides clear analytical results. Obviously, this comes at some cost in terms of realism of the underlying assumptions. In particular, we abstain from various frictions like market power and adjustments costs. Potentially important supply side details of alternative sources of energy are also ignored. Nevertheless, we think that the model provides useful results, also quantitatively. We are the first (to our knowledge) to analyze the intertemporal general equilibrium consequences of food-fuel competition in a setting with exhaustible resources. A transparent and tractable model, built on standard and well-known macroeconomic assumptions, may be particularly useful at such an early stage, leaving for future work the construction of models with less stylized assumptions solved using numerical techniques.

The paper is organized as follows. The next section describes formally our model and the relevant optimality conditions. In section 3, we show the existence of the distinct phases of fuel use. We derive expressions for balanced growth in the model and derive a sufficient condition for the economy to pass through the four phases in order 1 through 4, with the last state being absorbing. In Section 4, we analyze how much the price of food changes due to the increased food-fuel competition and present some calibrated simulations. In section 5, we add a carbon circulation model that dynamically determines the carbon concentration of the atmosphere and its impact on the global mean temperature. We then analyze the consequences of policies enacted in order to affect the use of biocarbon for fuel. Section 6 concludes.

2 A model of multiple use of biocarbon

We now describe a two-sector growth model with a manufacturing sector whose output can be consumed or invested and an agricultural sector producing biocarbon that serves as food and energy, energy being an input in the production of manufacturing goods. Manufacturing output Y_t is given by

$$Y_t = A_{1,t} K_{1,t}^{\alpha_1} L_{1,t}^{1-\alpha_1-\nu_1} E_t^{\nu_1} \quad (1)$$

where $A_{1,t}$ is an exogenous productivity trend, $K_{1,t}$ and $L_{1,t}$ are capital and labor and E_t is energy input. When prices later are defined, we use the manufacturing good as numeraire.

The agricultural sector produces biocarbon F_t according to the production function

$$F_t = A_{2,t} K_{2,t}^{\alpha_2} L_{2,t}^{(1-\alpha_2-\nu_2)} G^{\nu_2} \quad (2)$$

where $A_{2,t}$ is an exogenous productivity trend, $K_{2,t}$ and $L_{2,t}$ are capital and labor used in agriculture, and G is land, a fixed factor only used in agriculture. (Although we speak of an agricultural sector producing biocarbon for food or energy, sector 2 may be given a broader meaning, representing land-intensive production including e.g., wind and solar energy.)

Individual utility is given by

$$U = \sum_{t=0}^{\infty} \beta^t (\ln C_t + \theta \ln D_t) \quad (3)$$

where C_t is consumption of the manufactured good and D_t is consumption of biocarbon produced in the agricultural sector (food). The parameter θ determines the relative taste for food (more exactly, the taste for goods from sector 2), so that $\frac{\theta}{1+\theta}$ is the share of income spent on food.

We denote the flow of biocarbon that is used as energy input in sector 1 by B_t . Sector 1's energy input can also come from fossil carbon ("oil"), denoted by O_t , that is taken from a non-renewable and finite stock R_t without incurring extraction costs. B_t and O_t are perfect substitutes. It is key for our analysis that there be a one-sided substitutability between biocarbon and fossil carbon. While biocarbon can be used for energy services, fossil carbon cannot be eaten.

In addition to (1) and (2), the economy faces the following constraints:

$$\begin{aligned} Y_t &= C_t + K_{t+1} \\ F_t &= D_t + B_t \\ E_t &= O_t + B_t \\ K_t &= K_{1,t} + K_{2,t} \\ L_t &= L_{1,t} + L_{2,t} \\ B_t &\geq 0 \\ R_0 &\geq \sum_{t=0}^{\infty} O_t \\ O_t &\geq 0 \end{aligned} \quad (4)$$

The first constraint is the resource constraint for goods from sector 1. Manufacturing output is split between consumption and next period's capital stock. For analytical tractability, we assume that capital depreciates fully between periods. In our numerical examples, we will assume a period is 10 years, somewhat justifying the depreciation assumption.

The second equation states that output from agriculture (sector 2) is split between consumption and energy use. The third states that energy input stems from fossil carbon and the part of the biocarbon production that is not directly consumed as food. The fourth and fifth equations are, respectively, the aggregate resource constraints for capital and labor where the latter is exogenous. The sixth equation states that the use of biocarbon for energy has to be non-negative. This is a key restriction in the model. Allowing $B_t < 0$ would imply that $D_t > F_t$, i.e., that consumers eat fossil fuel. Finally, there are non-negativity constraints on the stocks and flows of fossil carbon. The possibility of using biocarbon as fuel creates a backstop technology. In contrast to the model derived by Dasgupta and Heal (1974), this implies the possibility that the non-negativity constraint on the flow of fossil fuel use ($O_t \geq 0$) binds at some t . As explained in the introduction, we assume that the technology becomes unexpectedly available at time 0, which characterizes the industrial revolution. We call the pre-industrial time, during which $O_t = 0$, phase 1.

We assume perfect markets so the planning allocation is identical to the competitive equilibrium.⁸ The planning allocation is found by maximizing (3) subject to the production functions (1), (2) and the resource constraints (4). The solution to the maximization problem provides a dynamically pareto-optimal allocation.

The Kuhn-Tucker formulation of the planner's problem can be written as the maximization of

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \{ \ln(A_{1,t}(K_t - K_{2,t})^{\alpha_1} (L_t - L_{2,t})^{\varepsilon_1} (O_t + B_t)^{\nu_1} - K_{t+1}) + \theta \ln(F_t - B_t) \} \\ & + \beta^t \{ \lambda_t^F (A_{2,t}(K_{2,t})^{\alpha_2} (L_{2,t})^{\varepsilon_2} G^{\nu_2} - F_t) + \lambda_t^B B_t + \lambda_t^O O_t \} + \lambda^R \left(R_0 - \sum_0^{\infty} O_t \right) \end{aligned} \quad (5)$$

where $\varepsilon_1 \equiv 1 - \alpha_1 - \nu_1$ and $\varepsilon_2 \equiv 1 - \alpha_2 - \nu_2$. The associated first-order conditions with

⁸The definition of a competitive equilibrium is standard and therefore omitted.

respect to $K_{t+1}, K_{2,t}, L_{2,t}, B_t, O_t$, and F_t , and the Kuhn-Tucker constraints are

$$\begin{aligned}
\frac{1}{C_t} &= \frac{\beta}{C_{t+1}} \frac{\alpha_1 Y_{t+1}}{K_{t+1} - K_{2,t+1}}, \\
\frac{1}{C_t} \frac{\alpha_1 Y_t}{K_t - K_{2,t}} &= \lambda_t^F \frac{\alpha_2 F_t}{K_{2,t}}, \\
\frac{1}{C_t} \frac{\varepsilon_1 Y_t}{L_t - L_{2,t}} &= \lambda_t^F \frac{\varepsilon_2 F_t}{L_{2,t}}, \\
\frac{1}{C_t} \frac{\nu_1 Y_t}{O_t + B_t} + \lambda_t^B &= \frac{\theta}{F_t - B_t}, \\
\frac{1}{C_t} \frac{\nu_1 Y_t}{O_t + B_t} + \lambda_t^O &= \beta^{-t} \lambda^R, \\
\frac{\theta}{F_t - B_t} &= \lambda_t^F, \\
\lambda_t^B B_t &= 0, \\
\lambda_t^O O_t &= 0.
\end{aligned} \tag{6}$$

The current-value Kuhn-Tucker multipliers λ_t^B and λ_t^O are strictly positive if the constraints $B_t \geq 0$ and $O_t \geq 0$ bind. The shadow value on the resource constraint for fossil fuel λ^R is strictly positive, since fossil fuel is always valuable for production and is assumed to be costless to extract.

3 Characterization of the allocation

We will now characterize the solution to the planner program. As noted above, the planner solution will coincide with the allocation in a standard decentralized competitive equilibrium. Therefore, we will use the notion of prices for relevant marginal products and marginal utilities.

We show that after the technology for using fossil fuel has been discovered, the economy will move through a series of distinct phases, determined by which of the Kuhn-Tucker multipliers λ_t^B and λ_t^O is binding. In one of the phases, only fossil fuel will be used and the price of biocarbon will then be higher than the price of fossil carbon. Therefore, only the multiplier λ_t^B will be strictly positive.

Second, there will a phase of mixed fuel use when both biocarbon and fossil carbon will be used as fuel. The share of biocarbon used as fuel will be determined such that the price of the two forms of fuel coincide. In this phase, both λ_t^B and λ_t^O will be zero.

Finally, there will be a phase when biocarbon will be the only source of fuel. Then $\lambda_t^O > 0$, while $\lambda_t^B = 0$. The interpretation of $\lambda_t^O > 0$ is that at any time period in this phase,

the price of fuel is strictly below the price that would have made it economically worthwhile to save oil for use in that period. This last phase will resemble the initial phase before the discovery of the fossil fuel technology. We derive analytical expressions for the savings rate, the share of biocarbon used as fuel and the endogenous allocation of capital and labor over the two sectors of production that apply in all phases.

After characterizing the different phases of the economy, we will show that under very mild and realistic conditions, the economy will pass through the different phases in a particular order. Therefore, we label the phases by the numbers 1 through 4, as noted in the introduction. The pre-industrial phase before the discovery of the fossil fuel technology is called phase 1. When the fossil technology is discovered, land, capital and labor used for the production of fuel can be released for other purposes. This reduces the scarcity of food, and the food price falls. However, the fossil fuel price is below the food price during phase 2, provided the stock of fossil fuel is sufficiently large.⁹ During phase 2, the fossil fuel price increases relative to the food price due to increased scarcity of the exhaustible stock of fossil carbon. Eventually, the fossil fuel price reaches the food price and phase 3 is entered. During this phase, the two prices rise jointly at the rate of interest. Under weak conditions this cannot continue forever. Intuitively, the existence of biocarbon as a backstop technology for fuel puts a cap on how much fuel prices can increase. Specifically, also if no fossil fuel is used, fuel prices are bounded since biocarbon can be used as fuel. This implies that eventually, all fossil reserves are exhausted and the economy enters the absorbing phase 4, when biocarbon is the single source of fuel. Abstracting from growth in technology and labor, the prices of food (biocarbon) and fuel develop schematically as shown in Figure 3.

⁹If not, the economy enters into phase 3 immediately.

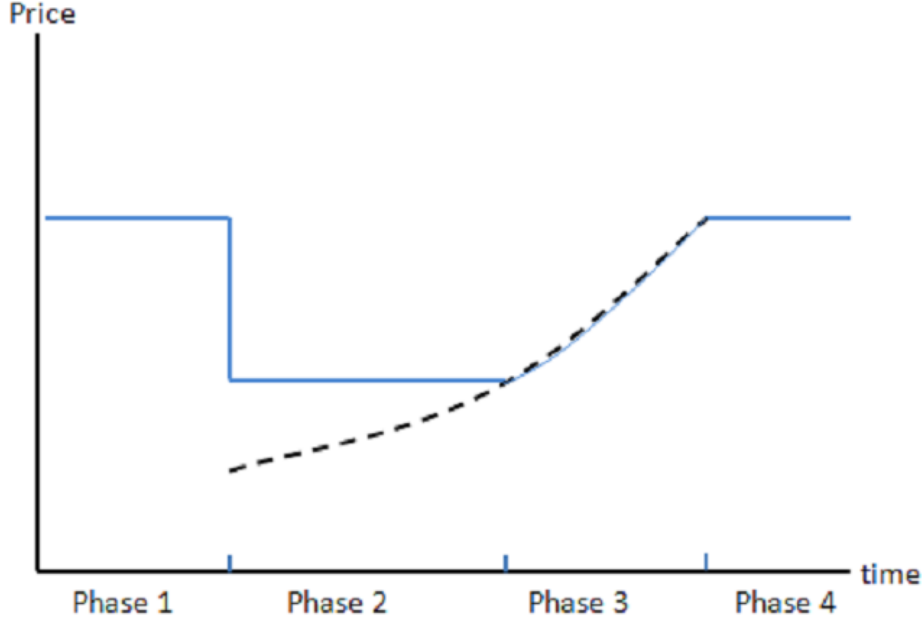


Figure 3. Schematic description of prices of biocarbon (solid) and fossil fuel (dashed).

3.1 Fuel phases

Let us begin this section by stating the following proposition formally.

Proposition 1 *After discovery of the fossil fuel technology, the economy passes through a maximum of three distinct phases;*

- *if $\lambda_t^B > 0$ and $\lambda_t^O = 0$, the economy is in phase 2 (Fossil) in period t . Fossil carbon is used ($O_t > 0$) but no biocarbon is burnt ($B_t = 0$).*
- *If $\lambda_t^B = \lambda_t^O = 0$, the economy is in phase 3 (Mixed) in period t . Fossil carbon is used ($O_t > 0$) and some biocarbon is used as fuel ($B_t > 0$).*
- *If $\lambda_t^B = 0$ and $\lambda_t^O > 0$, the economy is in phase 4 (Biocarbon) in period t . Fossil carbon is not used ($O_t = 0$) but biocarbon is burnt ($B_t > 0$).*

The economy has to be in phase 2 or 3 at least one period.

Proof in appendix.

Let us now characterize the endogenous dynamic allocation of capital and labor as well as the use of biocarbon over its potential uses as food and fuel. To this end, we define the share

of capital and labor used in biocarbon production as $\kappa_t \equiv \frac{K_{2,t}}{K_t}$ and $\Lambda_t \equiv \frac{L_{2,t}}{L_t}$, respectively, savings as $s_t \equiv \frac{Y_t - C_t}{Y_t}$, and the share of biocarbon used as fuel as $\Phi_t \equiv \frac{B_t}{F_t}$. Together with the use of fossil fuel, these four variables completely determine the endogenous dynamics of the model.

Using the first-order condition for F_t to substitute for λ_t^F in the first-order condition for $K_{2,t}$ and $L_{2,t}$ yields

$$\begin{aligned}\kappa_t &= \frac{1}{1 + \frac{(1-\Phi_t)\alpha_1}{\theta\alpha_2(1-s_t)}} \\ \Lambda_t &= \frac{1}{1 + \frac{(1-\Phi_t)\varepsilon_1}{\varepsilon_2(1-s_t)}}.\end{aligned}\tag{7}$$

Using the first line of (7) shifted one period ahead in the first-order condition for K_{t+1} yields

$$s_t = \beta \frac{\alpha_1(1 - \Phi_{t+1}) + \theta\alpha_2(1 - s_{t+1})}{\beta\alpha_1(1 - \Phi_{t+1}) + (1 - \Phi_{t+1} + \beta\theta\alpha_2)(1 - s_{t+1})}\tag{8}$$

The condition for B_t when $\lambda_t^B = 0$, yields

$$\nu_1 \frac{Y_t}{O_t + B_t} = \theta \frac{(1 - s_t)Y_t}{F_t(1 - \Phi_t)}.\tag{9}$$

Finally, the condition for O_t and O_{t+1} , provided $\lambda_{t+1}^O = \lambda_t^O = 0$, yields

$$\frac{O_{t+1} + B_{t+1}}{O_t + B_t} = \frac{\beta(1 - s_t)}{1 - s_{t+1}}\tag{10}$$

We can now use the first-order conditions to derive the following results.

Proposition 2 *Define the price of fuel as the marginal product of energy $\frac{\nu_1 Y_t}{E_t}$ and the price of biocarbon as the ratio of the marginal utility of food to the marginal utility of the manufactured good $\frac{\theta C_t}{F_t - B_t}$. In phase 2, the price of fuel grows at a rate identical to the marginal product of capital (the interest rate). The price of energy is below the price of food in phase 2, and in phases 3 and 4, the two prices are equal.*

Proof: In appendix.

The statement that the price of fuel grows at the interest rate is an implication of the famous Hotelling result. When both capital and the exhaustible resource can be used as a store of value, an interior solution requires the return on both to be the same.

We now show that in phases 2 and 4, there are balanced growth paths such that savings, capital and labor allocations and the use of fossil fuel are all constant.

Proposition 3 *In phases 2 and 4, there are balanced growth paths such that in phase 2*

$$\begin{aligned}\Phi_t &= 0, \\ s_t = s_F &\equiv \beta \frac{\alpha_1 + \theta \alpha_2}{1 + \beta \theta \alpha_2}, \\ \Lambda_t = \Lambda_F &\equiv \left(1 + \frac{\alpha_1}{\theta \alpha_2 (1 - s_F)}\right)^{-1}, \\ \kappa_t = \kappa_F &\equiv \left(1 + \frac{\varepsilon_1}{\theta \varepsilon_2 (1 - s_F)}\right)^{-1},\end{aligned}$$

and in phase 4,

$$\begin{aligned}\Phi_t = \Phi_B &\equiv \frac{\nu_1}{\nu_1 + \theta (1 - s_B)}, \\ s_t = s_B &\equiv s_F + \frac{\beta \alpha_2 \nu_1}{1 + \beta \theta \alpha_2}, \\ \Lambda_t = \Lambda_B &\equiv \left(1 + \frac{(1 - \Phi_B) \varepsilon_1}{\theta \varepsilon_2 (1 - s_B)}\right)^{-1}, \\ \kappa_t = \kappa_B &\equiv \left(1 + \frac{(1 - \Phi_B) \alpha_1}{\theta \alpha_2 (1 - s_B)}\right)^{-1}.\end{aligned}$$

Proof: Follows directly from equations (7)-(10).

Proposition 4 *If one or both of phases 2 and 4 are absorbing, their respective balanced growth paths are attained at the first period the economy is in the absorbing phase.*

Proof: The law-of-motion for the savings rate is unstable at any steady state with a root $\frac{ds_{t+1}}{ds_t} = \frac{1}{\alpha_1 \beta} > 1$.

We will later return to the important finding that the endogenous variables defined in proposition 3 are independent of the growth rates of the technology trends A_1 and A_2 .

3.2 Ordering of the phases

Let us now conclude this section by stating that when technological growth rates, $\gamma_{A_1,t}$ and $\gamma_{A_2,t}$ are constant,¹⁰ quite weak conditions are required for ruling out that phase 2 is absorbing while implying that phase 4 is absorbing.

Proposition 5 *If technological growth rates are constant and imply that $\gamma_F > \ln \beta$ in balanced growth, phase 2 is transitory while phase 4 is absorbing and always in balanced growth*

¹⁰We define the growth rate of a variable x as $\gamma_{x_t} \equiv \ln \left(\frac{x_t}{x_{t-1}} \right)$.

with a constant saving rate and constant allocation shares of capital, labor and biocarbon ($s_t = s_B$, $\kappa_t = \kappa_B$, $\Lambda_t = \Lambda_B$ and $\Phi_t = \Phi_B$).¹¹

Proof: In appendix.

Before discussing the intuition for the result, note that $\ln \beta < 0$, showing that the condition is weak.

To understand that phase 2 is transitory, note that the price of the fossil fuel has a growth rate given by the difference of the growth rate of manufacturing output γ_Y and the growth rate of fossil fuel use. The latter is given by equation (10), which here implies that $\gamma_O = \ln \beta$, which is negative. The growth rate of the price of food, instead, is given by the difference between the growth rate of manufacturing output and biocarbon output. Thus, food prices grow slower than fuel prices if $\gamma_F = \ln \beta$. When the fossil price grows faster than the biocarbon price, eventually the former must reach the latter and phase 2 ends. During phase 3 must the fuel price continues to rise, now together with the biocarbon (food) price. The price increase of biocarbon is driven by a falling use of fossil carbon that leads to a growing share of biocarbon used as fuel. However, this increase is bounded by the fact that also when no fossil fuel is used, the share of biocarbon used as fuel is interior implying a finite biocarbon price. Thuse, phase 3 is also transitory.

Finally, under exactly the same condition as made phase 2 transitory, the biocarbon price grows more slowly in phase 4 than the price that would be required for it to be worthwhile to store fossil fuel for use in some period in phase 4. Under these conditions, phase 4 is absorbing and fossil fuel will never be used again. All fossil fuel reserves are exhausted during phases 2 and 3, since it is not optimal to save some reserves that will never be used ($\lambda^R > 0$). Finally, we note that we can easily add the pre-industrial phase 1, in which fossil fuel (or the technology to use fossil fuel) has not yet been discovered. In key respects, it is identical to phase 4, where the fossil fuel is exhausted. In that sense, *the fossil era*, consisting of phases 2 and 3 (or only a phase 3), is a parenthesis, *just an episode in history*.

4 Quantitative implications

As acknowledged in the introduction, we are fully aware of the fact that our model is stylized and abstracts from potentially important aspects of reality, like frictions and the existence of other sources of energy, just to mention a few. Nevertheless, we believe that the model can be used for quantitative exercises, which is the purpose of this section. In particular, we will study how prices of food and fuel develop over time.

¹¹The proposition can easily be stated in terms of exogenous growth rates. See appendix.

4.1 Food-Fuel Competition

A key policy issue is how much the price of food will rise during the phase of rapid growth that occurs in phase 3. Let us now perform a simple exercise to calculate this. Let us first normalize the aggregate labor supply to unity and disregard population growth.¹² Also normalize the supply of land, G , to unity.

Clearly, the price ratio in general depends on how the capital stock and technology develop during the transition. Specifically, given the allocation variables (s, κ, Λ and Φ), the ratio of prices in period t' and t is proportional to

$$\frac{A_{1,t'}}{A_{1,t}} \left(\frac{A_{2,t}}{A_{2,t'}} \right)^{1-\nu_1} \left(\frac{K_{t'}}{K_t} \right)^{\alpha_1 - \alpha_2(1-\nu_1)}. \quad (11)$$

Thus, if $\gamma_{A_1} > (1 - \nu_1) \gamma_{A_2}$, technological developments tend to push food prices up. Similarly, if $\frac{\alpha_1}{1-\nu_1} > \alpha_2$, i.e., capital's share of income net of energy payments in manufacturing good production is higher than capital's income share in biocarbon production, capital accumulation tends to increase the food price by making biocarbon relatively more scarce. Let us now focus on the effect on food prices of the transition from phase 2 to 4, disregarding the direct effect of technology and capital accumulation. Thus, we hold technology and capital constant and focus on the effect on the food price by the change in demand for fuel due to the increase in the share of biocarbon used as energy input from zero to Φ_B .¹³

The food price in phase 2 depends on fossil fuel use. We thus consider the situation at the end of phase 2, when food and energy prices have become equal, implying

$$\frac{\theta(1-s_t)Y_t}{F_t} = \frac{\nu_1 Y_t}{O_t} \Rightarrow O_t = \frac{\nu_1 F_t}{\theta(1-s_t)}. \quad (12)$$

Using (12) and the production functions allows us to compute the ratio of food prices in phase 4 and at the end of phase 2. It is straightforward to show that (apart from the proportionality factor (11)) this ratio is given by

$$\frac{1}{1-\Phi_B} \left(\frac{1-\kappa_B}{1-\kappa_t} \right)^{\alpha_1} \left(\frac{\kappa_B}{\kappa_t} \right)^{-\alpha_2(1-\nu_1)} \left(\frac{1-\Lambda_B}{1-\Lambda_t} \right)^{\varepsilon_1} \left(\frac{\Lambda_B}{\Lambda_t} \right)^{-\varepsilon_2(1-\nu_1)} \frac{1-s_B}{1-s_t} \left(\frac{\theta(1-s_t)\Phi_B}{\nu_1} \right)^{\nu_1} \quad (13)$$

where s_t, κ_t and Λ_t denote the savings rate, capital and labor allocations at the end of phase

¹²Of course, population growth may itself be important for the development of food prices. But this is not the focus of this paper.

¹³Alternatively, assume $\gamma_{A_c} = (1 - \nu_1) \gamma_{A_f}$ and $\alpha_1 = (1 - \nu_1) \alpha_2$, in which case technology and capital have no effect on the food price.

2.¹⁴

Several things are worth noting in this expression. For reasonable parameters, the two last factors turn out to be close to unity. Comparing the steady state savings rates in proposition 3, we see that the savings rate increases by an amount smaller than $\beta \frac{\alpha_2 \nu_1}{1+\beta \theta \alpha_2}$ over the transition. Since ν_1 is on the order of a few percent, this is small. For the same reason the last factor is also close to unity. The first term $\frac{1}{1-\Phi_B}$ can then be interpreted as the direct effect on the food price of the increased fuel demand occurring during phase 3. If capital and labor are not allowed to relocate towards biocarbon production, all the four factors involving the allocation variables κ and Λ would be unity and the price increase would be given (approximately) by $\frac{1}{1-\Phi_B}$. We will later show that in our baseline benchmark calibration Φ_B will be around 0.71, i.e., in phase 4, 71% of biocarbon is used as fuel. Thus, in absence of endogenous capital and labor relocation, the increase in the demand for biocarbon to be used as fuel would imply that the price of food would increase by a factor $\frac{1}{0.29} \approx 3.45$, i.e., and increase by close to 250%!

The remaining four terms tend to mitigate the price increase. Since capital is moving towards biocarbon production, $\kappa_B > \kappa_t$. This reduces the price increase of food by making $\left(\frac{1-\kappa_B}{1-\kappa_t}\right)^{\alpha_1}$ and $\left(\frac{\kappa_B}{\kappa_t}\right)^{-\alpha_2(1-\nu_1)}$ smaller than unity. Similarly, moving labor to biocarbon production implies that $\Lambda_B > \Lambda_t$, makes the factors $\left(\frac{1-\Lambda_B}{1-\Lambda_t}\right)^{\varepsilon_1}$ and $\left(\frac{\Lambda_B}{\Lambda_t}\right)^{-\varepsilon_2(1-\nu_1)}$ smaller than unity to an extent determined by the labor shares.

Finally, we should note that the expression for the price increase in (13) implicitly determines the duration of phase 3. During phase 3, the price of biocarbon grows at the rate of interest. Given an interest rate, the price increase over phase 3 therefore determines the duration of phase 3. However, while the formula in (13) is independent of technological growth rates, clearly this is not the case for interest rates. Specifically, the interest rate tends to be higher the higher the growth rate of technology.

Let us now put parameter values to our expressions. We set $\alpha_1 = \alpha_2 = 0.3, \beta = 0.99$ ¹⁰ and study how the other parameters affect the price ratio. As a benchmark we set the income share of energy, ν_1 , to 0.05 and the income share of land, ν_2 , to 0.3. The parameter θ is set to match the world share of GDP in agriculture, which was 2.8% in 2010.¹⁵ Noting that the

¹⁴Inspection of the formulas for κ_t , κ_B , Λ_t , and Λ_B in (7) and proposition 3, shows that $\kappa_t < \kappa_F$ and $\Lambda_t < \Lambda_F$. This follows from the fact that $s_B > s_F$ and that the positive root in the law-of-motion for the savings rate implies a monotone convergence to the higher savings rate. However, in our calibration, we show that $\kappa_t \approx \kappa_B$ and $\Lambda_t \approx \Lambda_B$ to the third decimal point.

¹⁵Source: Worldbank, data downloaded from <http://data.worldbank.org/> October 8, 2012. The figure includes forestry.

Note also that this value is substantially smaller than the share of income spent on food in the national accounts. However, a large share of the final price of food is not related to agriculture but to service and transportation.

income share is $\theta/(1+\theta)$, this yields $\theta = 0.029$.

Since we do not have a closed form expression for s_t in the last period of phase 2, we cannot derive an exact expression for the price increase. However, we know that $s_B > s_t > s_F$ since the positive root and unstable root in the dynamic equation for savings imply monotone convergence. This provides a band for the potential price increase. Using $s_t = s_F$ gives a price increase of 40.1%, while if instead $s_t = s_B$ the price increase is 40.8%. We proceed the analysis under the approximation $s_t \approx s_F$, which also pins down κ_t and Λ_t from their formulas in (7) to κ_F and Λ_F .

As already noted, the direct effect, given by $\frac{1}{1-\Phi_B}$, implies a price increase of 243% since $\Phi_B = 0.7085$. The mitigation of this effect is largely due to the increase in κ from $\kappa_F = 2.05\%$ to $\kappa_B = 6.59\%$ and Λ from $\Lambda_F = 1.27\%$ to $\Lambda_B = 4.16\%$. The effect of these two factors alone through the factor $\left(\frac{\kappa_B}{\kappa_F}\right)^{-\alpha_2(1-\nu_1)} \left(\frac{\Lambda_B}{\Lambda_F}\right)^{-\varepsilon_2(1-\nu_1)}$ is to reduce the price increase to 56.8%.

Let us now do some sensitivity analysis. In the following graphs, we analyze how the effect of the transition on food prices responds to variation in the parameters. Specifically, we plot the price increase against ν_1 , θ and ν_2 , respectively, holding the other parameters at their benchmark values. Figure 4 shows that the price increase is increasing in the income share of energy in manufacturing production. This is not surprising, since a higher income share of energy leads to a larger increase in the demand for biocarbon as fossil fuel is exhausted. However, the sensitivity is (perhaps surprisingly) moderate – a doubling of the income share to 10% leads to a price increase of 57% rather than 40%.

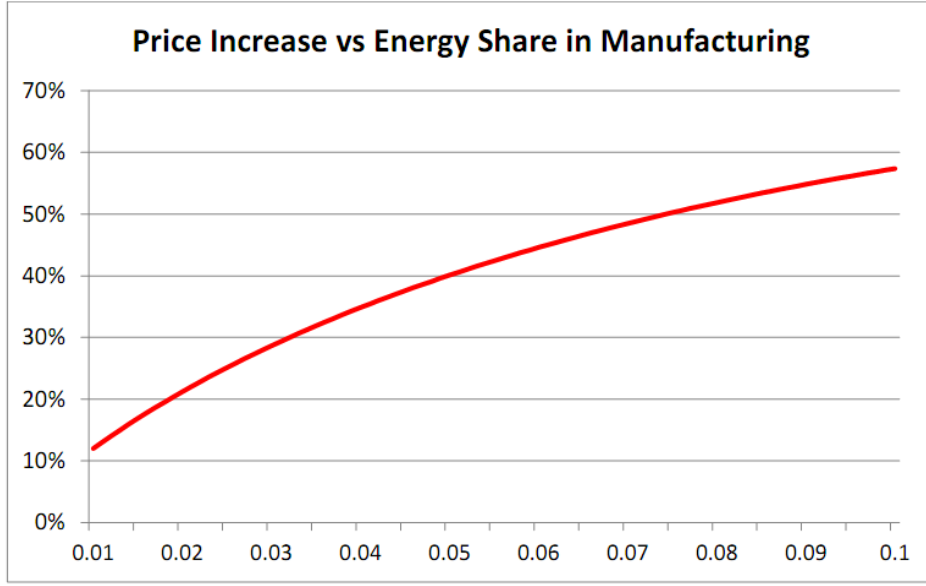


Figure 4. Price increase of fuel and biocarbon over phase 3 vs. energy income share in manufacturing (ν_1).

Figure 5 shows that the price increase falls in the income share of food. With a high income share of food, a large share of capital and labor is allocated to the biocarbon sector already in phase 2 and the extra increase in the demand for biocarbon therefore has small effects. At low income shares, the price increase can be quite substantial, implying a long transition period. If, for example, we calibrate θ to the US income share of agriculture of 1.2%, the price increase is a hefty 70.2%.

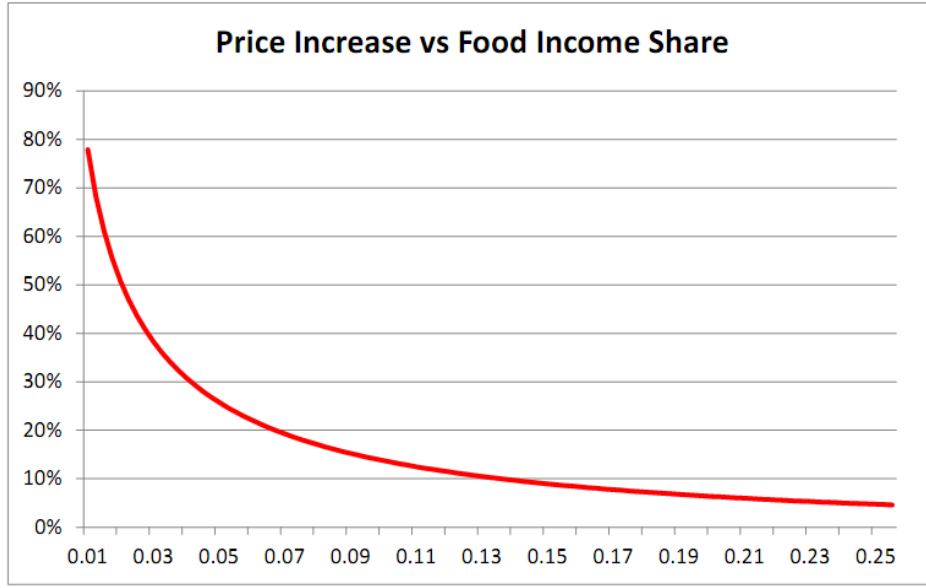


Figure 5. Price increase of fuel and biocarbon over phase 3 vs. income share of food ($\frac{\theta}{1+\theta}$).

Figure 6 shows the effect of varying the income share of land in agriculture. The price increase is higher the higher the income share of land. This is due to the fact that a higher income share of the fixed factor implies faster declining marginal products when output is increased by moving labor and capital to the sector. Although standard estimates of this income share are fairly low, we may consider higher values as reasonable if we think that additional reasons for a falling marginal product may exist.¹⁶ Here we are thinking of the fact that a higher demand for biocarbon leads to less productive land being taken into use.

¹⁶Land share in agriculture is 0.18 in Valentinyi and Herrendorf, (2007).

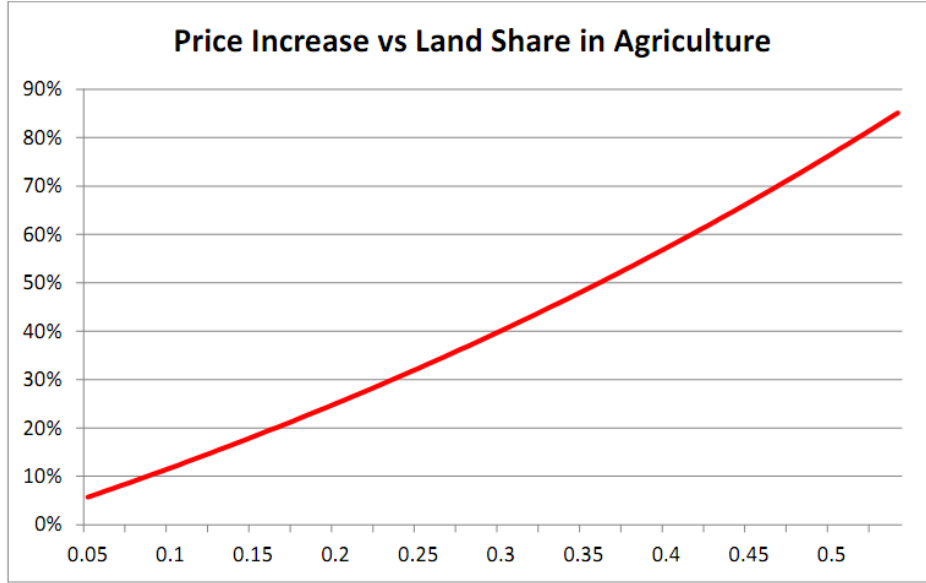


Figure 6. Price increase of fuel and biocarbon over phase 3 vs. income share of land in agricultural production (ν_2).

Finally, let us conclude by noting that the model is able to generate quite sizeable increases in the price of food if the income share of food is low and the energy and land shares are high. Setting $\theta = 0.012$, $\nu_1 = 0.08$, and $\nu_2 = 0.5$, which arguably is somewhat extreme, gives a price increase of 187%. In this case, over 90% of biocarbon production would be used as fuel in phase 4.

4.2 Price dynamics

Our model easily lends itself to calibrated simulations. In the appendix, we discuss the details of this. Here, we simply note that under the weak assumptions specified above, phase 4 is absorbing. Once phase 4 is reached, the economy evolves along the balanced growth path with a constant savings rate, constant capital and labor allocations and a constant share of biocarbon used for energy given by proposition 3. Thus, we can do backward induction from the beginning of phase 4. The balanced growth path in phase 2 is approached asymptotically backward in time.¹⁷

Figure 7 shows the evolution of energy prices and biocarbon prices. Both prices are normalized to be 100 at the start of phase 3. We set $\gamma_{A_1} = \gamma_{A_2} = 1.1$ per decade, $A_{1,0} = A_{2,0} = 1$ and K_0 so that the marginal product of capital is approximately constant at the beginning of the simulation period. Interest rates during the transition are then between

¹⁷However, since savings rates are not very different in phases 4 and 2, convergence is fairly rapid.

2.2% and 2.5% per year. The economy is in phase 2 until period 10. During this phase, energy prices grow at the rate of interest (on average by 2.3% per year) while food prices fall very slowly (about 0.05% per year). Phase 3 takes two periods and during this, prices of energy and biocarbon rise in parallel by 42% from the last period of phase 2 to the first period of phase 4.¹⁸ During phase 4, biocarbon prices increase slowly at close to 0.1% per year.

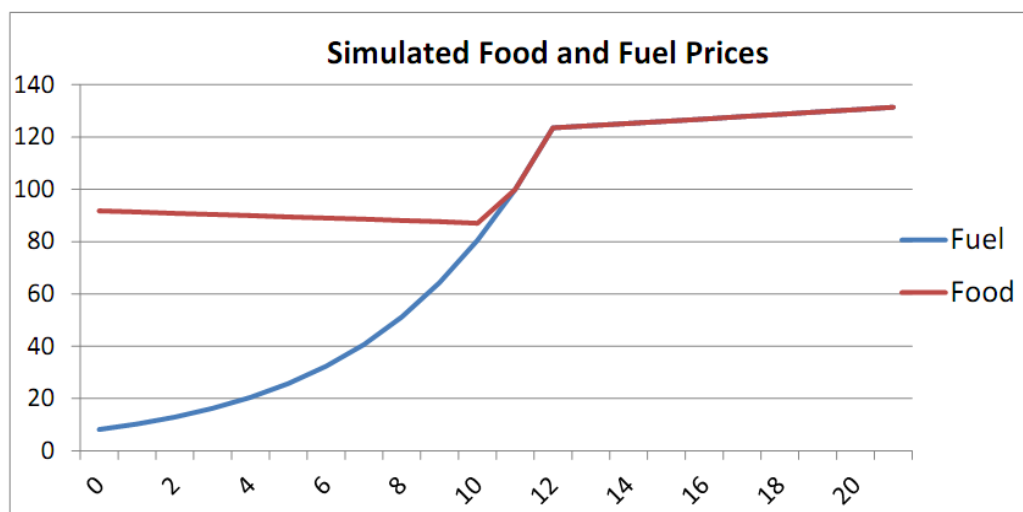


Figure 7. Prices of fuel and food over time.

5 Biocarbon and the climate

Proponents of biofuel argue that it provides a powerful possibility of slowing down global warming, and indeed biofuel is by far the most important green energy source in the OECD countries, accounting for 55% of all green energy used in the OECD countries.¹⁹ Antagonists, however, point out that biofuel has increased the food price and should therefore be banned. This section studies two kinds of constraints on the use of biocarbon, a total ban and an upper limit on the use of biocarbon as a source of fuel. To be able to infer implications for climate change, we append the model with a simple specification of a carbon cycle that determines the amount of CO₂ in the atmosphere as a function of the history of emissions. We use standard measures of how climate is affected by CO₂ concentration due to the greenhouse effect. We assume that the accumulation of CO₂ in the atmosphere affects the productivity

¹⁸The transition to the fossil fuel-free economy is likely to be longer in reality, of course. The discrepancy is due to the fact that we disregard fossil fuel sources with non-negligible extraction costs. We briefly discuss the consequences of this omission in the conclusion.

¹⁹See Sinn (2012, chapter 2).

trends A_1 and A_2 . In a decentralized economy, private choices will then be identical to the solution to the central planner solution where the externality is ignored (see Golosov et al., 2012 for more on the relation between the central planner solution and the decentralized equilibrium).²⁰

Let us first describe the restricted optimization problem. Here we will simply solve the program (5) under restrictions on the share of biocarbon that can be used as fuel. The first restriction is that the biofuel share in biocarbon production be zero: $\Phi_t = 0$ for all t . We will consider the consequences of having this condition being imposed unexpectedly at the end of phase 2, as this is where society is today, given that large-scale commercial biocarbon fuel use began only around 2005.

The second restriction we analyze is milder, namely $\Phi_t \leq \bar{\Phi}$ for all t when fossil carbon is still used. If $\bar{\Phi} > 0$, biocarbon will eventually replace fossil fuel and we allow the restriction to be lifted when all fossil fuel is used up. We label this *biocarbon phase-in*.

Solving the constrained problem is very easy. Under the constraint $\Phi_t = 0 \forall t$, the economy is simply in phase 2 forever, with $s_t = s_F$, $\Lambda_t = \Lambda_F$ and $\kappa_t = \kappa_F$ as defined in proposition 3. Fossil carbon use satisfies the Hotelling equation, which simplifies to $\frac{O_{t+1}}{O_t} = \beta$. Compared to the unconstrained case, the price of fossil carbon must increase when the constraint is unexpectedly introduced. The increase is necessary to reduce resource extraction at all points in time so as to exclude exhaustion in finite time. Given that the extraction of fossil carbon slows down, it is clear that the ban on biocarbon use as fuel will slow down climate change. Not a subsidy, but a ban on biofuel will be better for the climate. The explanation for the result can be sought in a variant of the Green Paradox (Sinn 2008, 2012), according to which measures that restrict the future demand for fossil fuel will accelerate global warming whereas measures that stimulate the future demand for fossil fuel will slow it down. In our model, phase 4, with its biocarbon-only regime, acts as an endogenously determined backstop technology. Banning this technology means increasing the future demand for fossil fuel, which provides an incentive to postpone the fossil carbon sales.

Under the second restriction, the economy goes through distinct phases as in the unconstrained case. If there is a sufficient amount of fossil fuel reserves still in ground when the restriction is introduced, the fossil fuel price is initially below the biocarbon price but grows over time at the rate of interest. Eventually, the price of fossil fuel reaches the biocarbon price and then phase 3 is reached and Φ_t becomes strictly positive, i.e., some biocarbon is

²⁰By specifying the climate externality we could easily find the optimal tax on fossil fuel using the optimal tax formula in Golosov et al (2012). A more ambitious task would be to solve a constrained optimization problem where we specify a climate externality but restrict the policy options of the planner so that the first best cannot be achieved. For example, we could ask the question of what is the optimal policy with respect to biocarbon use if the fossil carbon tax is set suboptimally? We leave this for future work.

used as fuel. The price of biocarbon and fossil carbon grow together at the rate of interest. Φ_t then increases over time and eventually $\Phi_t \leq \bar{\Phi}$ becomes binding. When this happens, equations (7), (8) and (10) hold with $\Phi_t = \bar{\Phi}$. This, together with equation (10), determines the dynamic path of the economy. When the constraint $\Phi_t \leq \bar{\Phi}$ binds, fuel prices are higher than food prices and the former continue to grow at the rate of interest. The fact that a strictly positive share $\bar{\Phi}$ of biocarbon production is allowed to be used for fuel implies that the fuel price cannot grow at the rate of interest forever. Therefore, fossil fuel will be fully exhausted in finite time. However, as the date of fossil carbon exhaustion is postponed relative to the unconstrained case, at each point in time there is less fossil carbon used, which again is an implication of the Green Paradox as in the case of a total ban.

By virtue of the numerical specifications given above, we are able to derive an estimate of the effect on temperature of the different policies. For this purpose, we follow the specification of the carbon cycle as given in Golosov et al. (2012), assuming a linear relation between fossil fuel use and the stock of atmospheric carbon.

$$S_t = \sum_{s=0}^{t+T} (1 - d_s) (O_{t-s} + X_{t-s}) \quad (14)$$

where $d_s \in [0, 1]$ for all s . Here, X_{t-s} is a sequence of exogenous emissions of greenhouse gas not coming from the use of O_t , and $1 - d_s$ represents the amount of carbon that is left in the atmosphere s periods into the future. As discussed in detail in Archer (2005) and Golosov et al (2012), a reasonable approximation to the carbon cycle is a process under which a share φ_L of carbon emitted into the atmosphere stays there forever, a share $1 - \varphi_0$ of the remainder quickly exits the atmosphere into the biosphere and the surface oceans, and the remaining part decays at a geometric rate φ . Such a process implies that $1 - d_s$ can be represented as

$$1 - d_s = \varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi)^s, \quad (15)$$

with $\varphi = 0.0228$, $\varphi_L = 0.2$, $\varphi_0 = 0.393$.

Regarding climate change, we rely on the logarithmic relation

$$Z_{t+1} = Z(S_t) = \lambda \ln \left(\frac{S_t}{\bar{S}} \right) / \ln 2, \quad (16)$$

where Z_t is global mean temperature above the pre-industrial steady state, $\bar{S} = 581$ GtC (Gigatons of carbon) is the pre-industrial atmospheric CO_2 concentration. This is a simplified version of the climate module used by Nordhaus (2008). The long-run response in Nordhaus' model corresponds to our direct effect. However, compared to Nordhaus, we abstract from

dynamic effects due to a drag on temperature induced by the fact that it takes longer to heat the oceans than to heat the atmosphere. However, except for quite extreme scenarios, these dynamics are not likely to be very important. A standard value for the climate sensitivity parameter λ here is 3.0 degrees Celsius. Thus, we assume that a doubling of the stock of atmospheric carbon leads to a 3-degree Celsius increase in the global mean temperature.

We now need to calibrate the model using units of fossil fuel in Gigatons of carbon. We assume that the current stock of fossil fuel reserves below ground is 300 Gt. As discussed in Golosov et al (2012), this is well in between current estimates of, on the one hand, estimates of existing oil that is economically profitable to extract at current economic and technical conditions, and on the other, estimates of total reserves.²¹ Since we have argued that it seems to be the case that the economy has already entered phase 3 by now, we choose the starting value of A_1 in order for this to occur at period zero in the simulation. In other words, in the simulation we think of period zero as the current decade (2010-2020). Furthermore, we set starting the value of CO_2 concentration to 802 GtC of which 103 is not depreciating (see Golosov et al, 2012). In order to capture other sources of greenhouse gas emissions, we set $X_t = 6 \text{ GtC/year}$ the first decade, thereafter 7.5 and then 10 for three decades. After that, exogenous emissions are assumed to fall to 7.5 GtC per year for three decades and then to 5.²²²³ We keep the other parameters unchanged and set (quite arbitrarily) $\bar{\Phi} = 0.3$.

In figure 8, we show the consequences of the global mean temperature of the three scenarios: laissez faire, biocarbon phase-in and biocarbon ban. We see that the two constrained policies lead to a substantial reduction in the speed of global warming.²⁴ During the coming decades, the biocarbon phase-in leads to a reduction in the temperature on order of one third degree Celsius, while the corresponding figure for the complete ban is about one-half degree. Although the long-run heating is the same in the three scenarios, postponing global warming may have a substantial social value.

²¹Recall that we do not model coal here. Coal exists in much larger supply and has a price that is, at least currently, largely determined by extraction costs rather than Hotelling rents. Including coal in the analysis is potentially important but left for future work.

²²IEA (2010) reports that global use of coal and gas in 2008 was 5.9 GtC.

²³The only effect of the exogenous carbon emission is that it slightly reduces the difference between the temperature impacts in the studied scenarios. This is due to the logarithmic relation between CO_2 concentration and temperature in (16), implying that the marginal effect of carbon emissions on temperature is lower at higher CO_2 concentrations.

²⁴Our model, as well as most other standard climate models, overpredict the current global mean temperature. A likely reason for this is the omission of the effect of short-lived aerosols that reflect incoming solar radiation. See Schwartz et al. (2010).

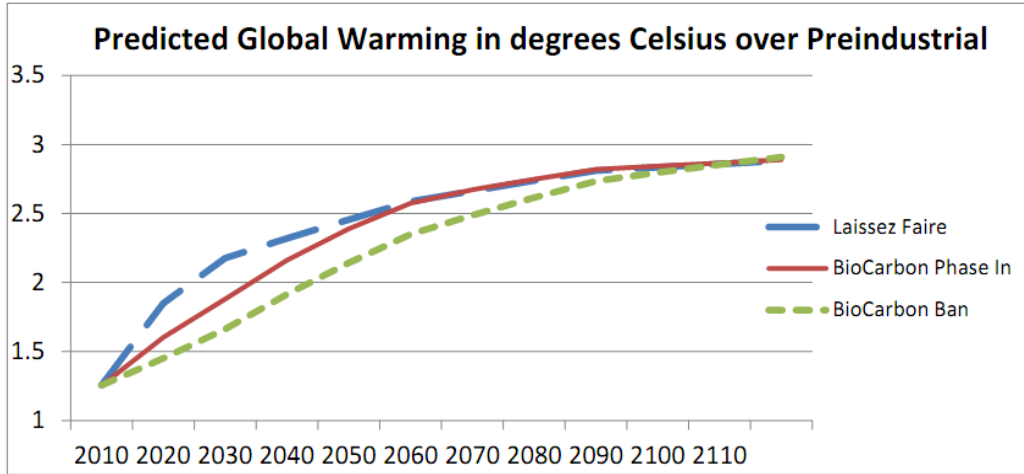


Figure 8. Global mean temperature over preindustrial level in degrees Celsius for three scenarios.

In Figure 9, we show the food price relative to the initial price prevailing at the switch from phase 2 to 3 under laissez faire. Under the complete ban, food prices are always lower than under laissez faire and actually fall over the whole simulation period. Under the biocarbon phase-in, the food price in the initial period is actually slightly higher than in laissez faire. The reason for this is that the expected future biocarbon burning ban leads to an current increase in the price of fuel. This in turn leads to an increase in Φ_{2010} up to $\bar{\Phi}$, implying higher food prices. After this fairly modest increase, food prices remain almost constant until the phase of mixed use of biocarbon and fossil fuel is over in 2070.

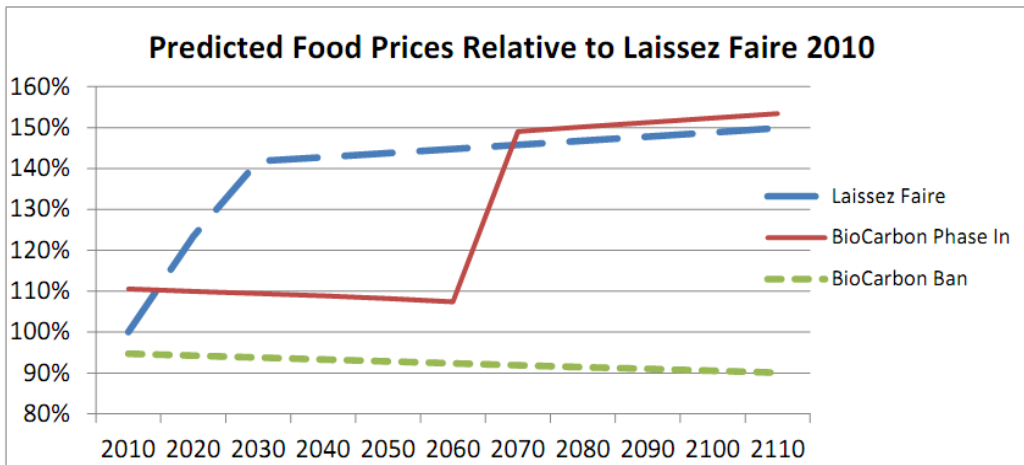


Figure 9. Food prices relative to price in 2010 under Laissez Faire for three scenarios.

Figure 10 shows output relative to laissez faire. Recall, however, that we have not specified the climate externality here. Thus, any gains in output due to the slower global warming are disregarded. The biocarbon ban leads quite substantial losses, in particular towards the end of the simulation, while the phase in leads to more modest losses. To analyze whether these losses are worth taking in order to achieve a slower global warming depends crucially on assumptions on the costs of climate change. We leave this for future work.

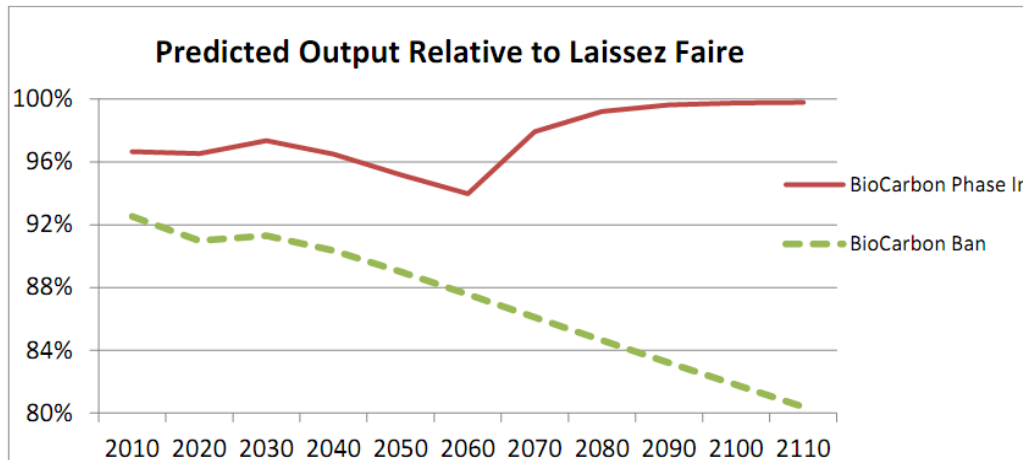


Figure 10. Manufacturing output relative to Laissez Faire for three scenarios.

6 Concluding remarks

In a nutshell, our theory of industrialization is simple. Since its very existence mankind has lived on biocarbon. Then, quite suddenly, a way to dig holes in the ground and use the fossil carbon stored there some 300 million years previously was found. This discovery was an integral part of rapid industrial growth and improved general levels of nutrition, as it meant that all of the arable land could be used for the production of food. When fossil carbon became scarcer mankind decided to reserve some of it for later use and return a share of the land to energy production. Finally, when fossil carbon is used up, mankind will have to go back to living on biocarbon only.

During the transition to the fossil-free economy, biocarbon becomes more scarce and food prices therefore increase. We have derived closed-form expressions for how much prices increase. We show, in particular, that a higher income share of energy in the production of consumption goods, a higher income share of land in the biocarbon production and a lower share of income spent by consumers on food leads to a higher increase in the price

of food during the transitional phase 3. Perhaps more importantly, we show that the price increase is quite sensitive to the possibility of increasing biocarbon output by moving factors of production to the biocarbon sector. Under the benchmark calibration, the price increase is about 40%, but if capital and labor cannot move, the implied price increase is on the order of 250%. This suggests that frictions to this relocation can be important quantitatively. We leave an analysis of this for future research.

In the current policy debate, policy makers have argued that subsidizing and stimulating the use of biocarbon is a way to reduce the speed of global warming. By contrast, we have shown that both a ban of biocarbon or a limit on the maximum share of biocarbon in total energy consumption would slow down climate change due to the mechanics of the Green Paradox. We also show that such restrictions can decouple food and fuel prices, at least for an extended period of time. The idea of subsidizing an increased use of biocarbon as fuel therefore seems to be bad in terms of climate change and potentially also bad in terms of increasing food prices.

To keep the model tractable and transparent we have abstracted from costs associated with the extraction of fossil fuel. This of course is not realistic, particularly not for coal. Coal exists in substantially larger quantities than cheap oil and its extraction costs are currently higher relative to price. Furthermore, the price of coal per unit of energy is much lower than for oil and biocarbon. We believe that the inclusion of stock-dependent extraction costs is a potentially interesting avenue for future research, but foresee that it will require sacrificing analytical tractability. This is why we have not included them in this paper.

We conjecture that adding extraction costs will increase the length of the transition phase 3 in which both fossil and biocarbon are used. This conjecture is due to the well-known fact that with stock-dependent extraction costs, the Hotelling-Solow-Stiglitz rule implies that the price increase relative to the price net of the unit extraction cost be equal to the rate of interest rate, which implies a rate of price increase below the rate of interest. With a lower rate of price increase, the transition phase between the fossil-carbon-only phase and the biocarbon-only-phase is longer.

Let us finish by shortly discussing distributional issues, obviously also absent from our representative agent framework. With the Tortilla crisis, the world economy may have entered phase 3. After a long period of fossil energy being cheaper than food, its price has finally risen to the level of the food price, pulling it along, enforcing massive reallocations of labor and capital from industry to agriculture and of food from the table to the tank. While all of this is Pareto optimal, apart from any global warming externality, we warn the reader that this does not, of course, mean that distributional goals are met.

Due to the homotheticity of the preferences assumed, the model is compatible with

an uneven allocation of property rights in land, capital, labor and the stock of the fossil fuel resource, provided everyone holds these endowments in equal proportions. However, although homotheticity is a convenient and standard assumption in growth theory, it may be an imperfect description of reality. Thus, if the price of food in phase 3 keeps rising, this will have more problematic implications for the well-being of individuals who spend a larger share of income on food. Generally, this is true of individuals in poor countries. We have shown that the food price increase during phase 3 is higher the lower the income share of biocarbon, and we have modeled this to be 2.9%, in line with the global average share of agriculture in GDP. With such a low income share, even substantial increases in food prices may have tolerable effects on welfare for the representative consumer. However, a large share of poor people in the world spend much more than 2.9% of their income on food and may be much more severely hurt by a food-price increase. Thus, in order to avoid the food price increase a ban on the use of biocarbon for energy purposes might be worth considering. Perhaps such a ban would generate the double dividend of helping the poor (and preventing further repetitions of the Tortilla crisis) and slowing down global warming. However, given that our model does not explicitly model income distribution, we can only touch on this problem and encourage researchers to investigate this issue in the future.

One of the questions to be asked in a distributional variant of our model would be which policy tools might serve the distributional goals, and what welfare losses in the Pareto sense such tools would involve, if any. There is a broad consensus that redistribution via cash flow taxes or, equivalently, a once-and-for-all reallocation of endowments such as land ownership titles would, if unforeseen, be possible without incurring welfare losses. It would be useful in our opinion to study whether land reforms giving the poor entitlements to land might not be a practical way to solve the redistribution problem, without sacrificing overall economic efficiency and welfare.

7 References

Ajanovic, A. (2010), "Biofuels versus Food Production: Does Biofuels Production Increase Food Prices?," *Energy* 2010, forthcoming, available online: doi:10.1016/j.energy.2010.05.019.

Archer, David (2005), "The Fate of Fossil Fuel CO₂ in Geologic Time", *Journal of Geophysical Research*, 110(C9).

Braun, J. von (2008), High Food Prices: The What, Who and How Proposed Policy Actions, Policy Brief, International Food Policy Research Institute (IFPRI), Washington DC.

Collins, K. (2008), "The Role of Biofuels and other Factors in Increasing Farm and Food Prices: A Review of Recent Developments with a Focus on Feed Grain Markets and Market Prospects," Report commissioned by Kraft Food Global, June 19, 2008,

http://www.globalbioenergy.org/uploads/media/0806_Keith_Collins_-_The_Role_of_Biofuels_and_Other_Factors.pdf.

Dasgupta, P., and G. Heal (1974), "The Optimal Depletion of Exhaustable Resources," *Review of Economic Studies* 41, p. 3 - 28.

EIA (2012), "Key World Energy Statistics," International Energy Agency.

Gilbert, C. L. (2010), "How to Understand High Food Prices," *Journal of Agricultural Economics*, 61, p. 398 – 425.

Golosov, A., J. Hassler, P. Krusell and A. Tsyvinski (2012), "Optimal Taxes on Fossil Fuel in General Equilibrium," NBER WP 17348.

Headey, D., and S. Fan (2008), "Anatomy of a Crisis: The Causes and Consequences of Surging Food Prices," *Agricultural Economics* 39, 2008, supplement, p. 375-391.

Herrendorf, B., and A. Valentinyi (2008), "Measuring Factor Income Shares at the Sector Level." *Review of Economic Dynamics*, 11, 820-835.

Hotelling, H. (1931), "The Economics of Exhaustible Resources", *Journal of Political Economy* 39, p. 137-175.

Mitchell, D. (2008), "A Note on Rising Food Prices," World Bank Working Paper No. 4682, July 2008.

Nordhaus, W. (2008), *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press, New Haven, CT.

Penn State, College of Agricultural Sciences, (2012), <http://extension.psu.edu/energy/field-crops/shelled-corn/heat-energy-content-of-shelled-corn>, downloaded at Nov. 20, 2012.

Piesse, J., and C. Thirtle (2009), "Three Bubbles and a Panic: An Explanatory Review of Recent Food Commodity Price Events," *Food Policy* 34, p. 119 - 129.

Rosegrant, M. W. (2008), International Food Policy Research Institute (IFPRI), Biofuels

and Grain Prices: Impacts and Policy Responses, Testimony for the U.S. Senate Committee on Homeland Security and Governmental Affairs, 7 May, 2008.

Schwartz, Stephen, Robert Charlson, Ralph Kahn, John Ogren, and Henning Rodhe (2010), “Why Hasn’t Earth Warmed as Much as Expected?”, *Journal of Climate*, 23.

Sinn, H.-W. (2008), “Public Policies against Global Warming: A Supply Side Approach,” *International Tax and Public Finance* 15, p. 360–394.

Sinn, H.-W. (2012), *The Green Paradox. A Supply Side Approach to Global Warming*, MIT Press, Cambridge, Mass.

8 Appendix

Proof of proposition 1

We first show that $\lambda_t^B \lambda_t^O = 0$. To do this, suppose otherwise that both λ_t^B and $\lambda_t^O > 0$, then $B_t = O_t = 0$, implying $Y_t = 0 = C_t$ and therefore utility is minus infinity. This is not part of an optimal plan, provided $K_0 > 0$. Then, to show that $\lambda_t^O = 0$ for some t , suppose otherwise that $\lambda_t^O > 0 \forall t$, then $O_t = 0 \forall t$. Then utility could be increased by setting $O_t = R_0$ at any t .

Proof of proposition 2

Suppose the economy is in phase 2 in period t and $t + 1$. Then, the ratio of the first-order conditions for O_{t+1} and O_t can be written

$$\frac{\nu_1 Y_{t+1} E_{t+1}^{-1}}{\nu_1 Y_t E_t^{-1}} = \frac{C_{t+1}}{\beta C_t}$$

where the LHS is the growth rate of the price of energy. From the first order condition for K_{t+1} we get the standard Euler equation

$$\frac{C_{t+1}}{C_t \beta} = \frac{\alpha_1 Y_{t+1}}{K_{t+1} - K_{2,t+1}} \quad (17)$$

where the RHS is the gross interest rate. Clearly, this is simply a variant of the Solow-Stiglitz efficiency condition, which because of the assumption of competitive markets coincides with the Hotelling condition (Hotelling, 1931). Regarding the second claim made in the proposition, the first-order conditions for B_t implies

$$\frac{\theta C_t}{F_t - B_t} - \frac{\nu_1 Y_t}{O_t + B_t} = C_t \lambda_t^B$$

where the LHS is the difference between the price of biocarbon and energy and the RHS is positive by definition if the economy is in phase 2, while it is zero in phases 3 and 4.

Proof of proposition 5

Suppose that phase 2 were absorbing. In such a case, the economy would necessarily be on a balanced growth path as described in proposition 3. Then, the growth rate of the manufacturing output is

$$\gamma_Y = \frac{\gamma_{A_1} + \varepsilon_1 \gamma_L + \nu_1 \ln \beta}{1 - \alpha_1} \quad (18)$$

and the growth rate of biocarbon production is given by

$$\gamma_F = \gamma_{A_2} + \alpha_2 \gamma_Y + \varepsilon_2 \gamma_L. \quad (19)$$

The price of fossil fuel is determined by the ratio $\frac{\nu_1 Y_t}{O_t}$. Since by (10), the growth rate of fossil fuel use is $\ln \beta$, the growth rate of the fossil fuel price is $\gamma_Y - \ln \beta$, i.e., the growth rate of output plus the subjective discount rate. The growth rate of the food price ($\frac{\theta C_t}{F_t}$) is given by the difference $\gamma_Y - \gamma_F$, since C grows at the same rate as Y . Thus, the growth rate of fossil fuel price is larger than the growth rate of the food price if $-\ln \beta > -\gamma_F$. Thus, as long as biocarbon production is not *falling* at a rate higher than the subjective discount rate, fossil fuel prices grow faster than food prices. Since phase 2 requires that fossil fuel prices are below food prices, $-\ln \beta > -\gamma_F$ is a sufficient condition for phase 2 not being absorbing.²⁵

Now, consider phase 4. The first-order condition for O_t can be written

$$\frac{\nu_1 Y_t}{B_t} = C_t \beta^{-t} \lambda^R - C_t \lambda_t^O$$

where $\lambda_t^O > 0$ by definition of phase 4. The LHS is the energy price and the first term on the RHS is the current shadow value of the resources constraint on fossil fuel ($\beta^{-t} \lambda^R$) divided by marginal utility to express it in terms of the manufacturing good. The term $C_t \beta^{-t} \lambda^R$ is to be interpreted as the *price of energy required* for it to be worthwhile to save fossil fuel for use in period t . If the actual price (LHS) is equal to (below) this, $\lambda_t^O = 0$ (> 0). In phase 4, $\lambda_t > 0$ implying that the actual energy price is below the threshold required for it to save fossil fuel until period t .

Now, let us consider what is required for us to conclude that if the economy is in phase 4 in period t , it is also in phase 4 in period $t + 1$. A sufficient condition for this is that $\frac{\nu_1 Y_t}{B_t}$ grows slower than $C_t \beta^{-t} \lambda^R$, i.e., that $\frac{Y_{t+1}}{Y_t} \frac{B_t}{B_{t+1}} < \frac{C_{t+1}}{\beta C_t}$. The RHS of this inequality is equal to the marginal product of capital (the interest rate) by the Euler equation (17). From the

²⁵It is immediate to express this condition in terms of exogenous parameters. Using (18) and (19) we calculate $\gamma_F = \frac{\gamma_{A_1}(1-\alpha_1) + \alpha_2 \gamma_{A_2} + (\alpha_2 \varepsilon_1 + \varepsilon_2(1-\alpha_1))\gamma_L + \alpha_2 \nu_1 \ln \beta}{1-\alpha_1}$. Clearly, non-negative growth rates of technology and labor are sufficient for $\gamma_F > 0 > \ln \beta$.

Hotelling condition, we know that in order for it to be worthwhile to save fossil fuel, its price must grow at the interest rate. Thus, if the energy price is growing at a lower rate, it is not worthwhile to save fossil fuel. Using the fact that the savings rate and the share of biocarbon burned are constant, this simplifies to

$$-\ln \beta > -\gamma_F$$

i.e., the same condition as by which we could rule out an absorbing phase 2.²⁶

8.1 Simulation

Our model easily lends itself to calibrated simulations. The fact that under the weak assumptions specified above phase 4 is absorbing, implies that we can do backward induction from that phase back to phase 2. The simulation is easily done in a spreadsheet program and an example is available from the authors on request. Once phase 4 is reached, the economy evolves along the balanced growth path with the constant savings rate, constant capital and labor allocations and constant share of biocarbon used for energy given by proposition 3

Specifically, let us denote by T the first period in phase 4. We can then determine savings in the previous period by using (8). Clearly, this implies $s_{T-1} = s_B$.

We then conjecture that phase 3 is at least one period long. The exact oil use in this period will depend on initial conditions. We consider the case when remaining oil is almost zero. The condition that the price of food equals the price of energy then simplifies when O_{T-1} approaches zero from above $\Phi_{T-1} \rightarrow \Phi_B$. Again using (8), we find that $s_{T-2} = s_B$ as well. However, clearly O_{T-2} is not zero but must instead satisfy the Hotelling condition (10), here implying

$$\begin{aligned} \frac{B_{T-1}}{\beta} &= O_{T-2} + B_{T-2}, \\ \frac{\Phi_B F_{T-1}}{\beta} &= O_{T-2} + \Phi_{T-2} F_{T-2} \end{aligned} \tag{20}$$

Using this equation and conjecturing that period $T - 2$ is in phase 3, so that the price of

²⁶Again, this is easily expressed in exogenous parameters. The growth rate of biocarbon production as $\gamma_F = \frac{\gamma_{A_f}(1-\alpha_1) + \alpha_2\gamma_{A_c} + (\alpha_2\varepsilon_1 + \varepsilon_2(1-\alpha_1))\gamma_L}{1-\alpha_1-\alpha_2\nu_1}$ which is larger than the growth rate calculated in the previous footnote.

food equals that of energy, we get

$$\begin{aligned}\frac{\nu_1}{O_{T-2} + \Phi_{T-2}F_{T-2}} &= \frac{\theta(1-s_B)}{F_{T-2}(1-\Phi_{T-2})} \\ \frac{\frac{\nu_1}{\Phi_B F_{T-1}}}{\beta} &= \theta \frac{(1-s_B)}{F_{T-2}(1-\Phi_{T-2})} \\ 1 - \Phi_{T-2} &= \frac{F_{T-1}}{F_{T-2}} \frac{\theta(1-s_B)\Phi_B}{\nu_1\beta}\end{aligned}$$

Noting that κ_{T-2} and Λ_t are determined by Φ_{T-2} since $s_{T-2} = s_B$ this is an equation in Φ_{T-2} only, given K_{T-2} . Since it turns out that savings rates are not moving much for the calibrated parameters, it is easy to find a guess for K_{T-2} from the balanced growth path.²⁷ This gives us Φ_{T-2} and all endogenous variables in period $T-2$. Our calibration implies that $T-3$ is in phase 2, which is verified by comparing the food and fuel prices in period $T-3$. Therefore, the backward recursion on savings is now straightforward:

$$\begin{aligned}s_{T-3} &= \beta \frac{\alpha_1(1-\Phi_{T-2}) + \theta\alpha_2(1-s_B)}{\beta\alpha_1(1-\Phi_{T-2}) + (1-\Phi_{T-2} + \beta\theta\alpha_2)(1-s_B)}, \\ s_{T-s} &= \beta \frac{\alpha_1 + \theta\alpha_2(1-s_{T-s+1})}{\beta\alpha_1 + (1 + \beta\theta\alpha_2)(1-s_{T-s+1})} \forall s > 3,\end{aligned}$$

and for oil use

$$\begin{aligned}O_{T-3} &= (O_{T-2} + B_{T-2}) \frac{1-s_{T-2}}{\beta(1-s_{T-3})}, \\ O_{T-s} &= O_{T-s+1} \frac{1-s_{T-s+1}}{\beta(1-s_{T-s})} \forall s > 3.\end{aligned}$$

By iterating backward, this gives us a complete sequence of savings rates. Together with (7) this allows us to construct the complete series of all endogenous variables and thus provides a check that the initial guess on K_{T-2} was accurate. Otherwise we simply replace it. The procedure described above is easily implemented in a spreadsheet program.²⁸

²⁷When all savings rates are found, we construct the capital stocks from initial conditions.

²⁸An example is available upon request from the authors.